Chapter 01

Number System

A system in which we study different types of numbers, their relationship and rules govern in them is called as **number system.**

In the Hindu-Arabic system, we use the symbols 0, 1, 2, 3, 4, 5, 6, 7, 8 and 9. These symbols are called **digits.** Out of these ten digits, 0 is called an **insignificant digit** whereas the others are called **significant digits.**

Numerals

A mathematical symbol representing a number in a systematic manner is called a numeral represented by a set of digits.

How to Write a Number

To write a number, we put digits from right to left at the places designated as units, tens, hundreds, thousands, ten thousands, lakhs, ten lakhs, crores, ten crores.

Let us see how the number 308761436 is denoted

It is read as

Ten crores	Crores	Ten lakhs	Lakhs	Ten thousands	Thousands	Hundreds	Tens	Units
10^8	10^7	10^6	10^5	10^4	10^3	10^2	10^1	10^0
3	0	8	7	6	1	4	3	6

Thirty crore eighty seven lakh sixty one thousand four hundred and thirty six.

Face Value and Place Value of the Digits in a Number

Face Value

In a numeral, the face value of a digit is the value of the digit itself irrespective of its place in the numeral.

For example In the numeral 486729, the face value of 8 is 8, the face value of 7 is 7, the face value of 6 is 6, the face value of 4 is 4, and so on.

Place Value (or Local Value)

In a numeral, the place value of a digit changes according to the change of its place. *Look at the following to get the idea of place value of digits in* 72843016.

Crores 7 →	Place value of 7 →	$7 \times 10000000 = 70000000$
Ten Lakhs 2 →	Place value of 2 →	$2 \times 1000000 = 2000000$
Lakhs 8 →	Place value of 8 →	$8 \times 100000 = 800000$
Ten Thousands 4 →	Place value of 4 →	$4 \times 10000 = 40000$
Thousands 3 →	Place value of 3 →	$3 \times 1000 = 3000$
Hundreds 0 →	Place value of 0 →	$0 \times 100 = 0$
Tens 1 →	Place value of 1 →	$1 \times 10 = 10$
Units 6 →	Place value of 6 →	$6 \times 1 = 6$

It is clear from the above presentation that to obtain the place value of a digit in a numeral, we multiply the digit with the value of its place in the given numeral.

Types of Numbers

1. Natural Numbers

Natural numbers are counting numbers. They are denoted by N.

For example $N = \{1, 2, 3, \dots\}$.

- All natural numbers are positive.
- Zero is not a natural number. Therefore, 1 is the smallest natural number.

2. Whole Numbers

All natural numbers and zero form the set of whole numbers. Whole numbers are denoted by W.

For example $W = \{0, 1, 2, 3, \dots\}$

- Zero is the smallest whole number.
- Whole numbers are also called as non-negative integers.

3. Integers

Whole numbers and negative numbers form the set of integers. They are denoted by I.

For example $I = \{\dots, -4, -3, -2, -1, 0, 1, 2, 3, 4, \dots\}$

Integers are of two types.

(i) **Positive Integers** Natural numbers are called as positive integers. They are denoted by I^+.

For example $I^+ = \{1, 2, 3, 4, \dots\}$

(ii) **Negative Integers** Negative of natural numbers are called as negative integers. They are denoted by I^-.

For example $I^- = \{-1, -2, -3, -4, \dots\}$

- '0' is neither +ve nor –ve integer.

4. Even Numbers

A counting number which is divisible by 2, is called an even number.

For example 2, 4, 6, 8, 10, 12, ... etc.

- The unit's place of every even number will be 0, 2, 4, 6 or 8.

5. Odd Numbers

A counting number which is not divisible by 2, is known as an odd number.

For example 1, 3, 5, 7, 9, 11, 13, 15, 17, 19, ... etc.

- The unit's place of every odd number will be 1, 3, 5, 7 or 9.

6. Prime Numbers

A counting number is called a prime number when it is exactly divisible by, 1 and itself.

For example 2, 3, 5, 7, 11, 13, ... etc.

- 2 is the only even number which is prime.
- A prime number is always greater than 1.
- 1 is not a prime number. Therefore, the lowest odd prime number is 3.
- Every prime number greater than 3 can be represented by $6n + 1$, where n is integer.

How to test a Number is prime or not?

If P = Given number, then

(i) Find whole number x such that $x > \sqrt{P}$.

(ii) Take all the prime numbers less than or equal to x.

(iii) If none of these divides P exactly, then P is prime otherwise P is non-prime.

For example Let $P = 193$, clearly $14 > \sqrt{(193)}$

Prime numbers upto 14 are : 2, 3, 5, 7, 11, 13.

No one of these divides 193 exactly.

Hence, 193 is a prime number.

7. Composite Numbers

Composite numbers are non-prime natural numbers. They must have atleast one factor apart from 1 and itself.

For example 4, 6, 8, 9, etc.

- Composite numbers can be both odd and even.
- 1 is neither a prime number nor composite number.

8. Coprimes

Two natural numbers are said to be coprimes, if their HCF is 1.

For example (7, 9), (15, 16)

- Coprime numbers may or may not be prime.

9. Rational Numbers

A number that can be expressed as p/q is called a rational number, where p and q are integers and $q \neq 0$.

For example $\frac{3}{5}, \frac{7}{9}, \frac{8}{9}, \frac{13}{15}$ etc.

10. Irrational Numbers

The numbers that cannot be expressed in the form of p/q are called irrational numbers, where p and q are integers and $q \neq 0$.

For example $\sqrt{2}, \sqrt{3}, \sqrt{7}, \sqrt{11}$ etc.

- π is an irrational number as 22 / 7 is not the actual value of π but it is its nearest value.
- Non-periodic infinite decimal fractions are called as irrational number.

11. Real Numbers

Real numbers include rational and irrational numbers both.

For example $\frac{7}{9}, \sqrt{2}, \sqrt{5}, \pi, \frac{8}{9}$ etc.

- Real numbers are denoted by R.

Operations on Numbers

Addition

When two or more numbers are combined together, then it is called addition. Addition is denoted by '+' sign.

For example $24 + 23 + 26 = 73$

Subtraction

When one or more numbers are taken out from a larger number, then it is called subtraction.

Subtraction is denoted by '–' sign.

For example $100 - 4 - 13 = 100 - 17 = 83$

Division

When D and d are two numbers, then $\frac{D}{d}$ is called the operation of division, where D is the **dividend** and d is the **divisor**. A number which tells how many times a divisor (d) exists in dividend D is called the **quotient** Q.

If dividend D is not a multiple of divisor d, then D is not exactly divisible by d and in this case **remainder** R is obtained.

Let us see the following operation of division

Let $D = 17$ and $d = 3$

Then, $\frac{D}{d} = \frac{17}{3} = 5\frac{2}{3}$

Here, $5 =$ Quotient (Q),

$3 =$ Divisor (d)

and $2 =$ Remainder (R)

We see,

$$3 \text{ (Divisor)} \times 5 \text{ (Quotient)} + 2 \text{ (Remainder)} = 17 \text{ (Dividend)}$$

Hence, we can write a formula,

$$\text{Dividend} = (\text{Divisor} \times \text{Quotient}) + \text{Remainder}$$

Multiplication

When 'a' is multiplied by 'b', then 'a' is added 'b' times or 'b' is added 'a' times. It is denoted by '×'.

Let us see the following operation on Multiplication

If $a = 2$ and $b = 4$, then $2 \times 4 = 8$

or

$(2 + 2 + 2 + 2) = 8$

Here, 'a' is added 'b' times or in other words 2 is added 4 times.

Similarly, $4 \times 2 = 8$ or $(4 + 4) = 8$

In this case, 'b' is added 'a' times or in other words 4 is added 2 times.

Divisibility Tests

Divisibility by 2 When the last digit of a number is either 0 or even, then the number is divisible by 2.

For example 12, 86, 472, 520, 1000 etc., are divisible by 2.

Divisibility by 3 When the sum of the digits of a number is divisible by 3, then the number is divisible by 3. *For example*

(i) 1233 $1+2+3+3=9$, which is divisible by 3, so 1233 must be divisible by 3.

(ii) 156 $1+5+6=12$, which is divisible by 3, so 156 must be divisible by 3.

Divisibility by 4 When the number made by last two-digits of a number is divisible by 4, then that particular number is divisible by 4. Apart from this, the number having two or more zeroes at the end, is also divisible by 4. *For example*

(i) 6428 is divisible by 4 as the number made by its last two digits *i.e.*, 28 is divisible by 4.

(ii) The numbers 4300, 153000, 9530000 etc., are divisible by 4 as they have two or more zeroes at the end.

Divisibility by 5 Numbers having 0 or 5 at the end are divisible by 5. *For example* 45, 4350, 135, 14850 etc., are divisible by 5 as they have 0 or 5 at the end.

Divisibility by 6 When a number is divisible by both 3 and 2, then that particular number is divisible by 6 also.

For example 18, 36, 720, 1440 etc., are divisible by 6 as they are divisible by both 3 and 2.

Divisibility by 7 A number is divisible by 7 when the difference between twice the digit at ones place and the number formed by other digits is either zero or a multiple of 7.

For example 658 is divisible by 7 because $65-2\times 8=65-16=49$. As 49 is divisible by 7, the number 658 is also divisible by 7.

Divisibility by 8 When the number made by last three digits of a number is divisible by 8, then the number is also divisible by 8. Apart from this, if the last three or more digits of a number are zeroes, then the number is divisible by 8. *For example*

(i) 2256 As 256 (the last three digits of 2256) is divisible by 8, therefore 2256 is also divisible by 8.

(ii) 4362000 As 4362000 has three zeroes at the end. Therefore it will definitely divisible by 8.

Divisibility by 9 When the sum of all the digits of a number is divisible by 9, then the number is also divisible by 9. *For example*

(i) 936819 $9+3+6+8+1+9=36$ which is divisible by 9. Therefore, 936819 is also divisible by 9.

(ii) 4356 $4+3+5+6=18$ which is divisible by 9. Therefore, 4356 is also divisible by 9.

Divisibility by 10 When a number ends with zero, then it is divisible by 10.

For example 20, 40, 150, 123450, 478970 etc., are divisible by 10 as these all end with zero.

Divisibility by 11 When the sums of digits at odd and even places are equal or differ by a number divisible by 11, then the number is also divisible by 11. *For example*

(i) 2865423 Let us see

Sum of digits at odd places $(A) = 2 + 6 + 4 + 3 = 15$

Sum of digits at even places $(B) = 8 + 5 + 2 = 15 \Rightarrow A = B$

Hence, 2865423 is divisible by 11.

(ii) 217382 Let us see

Sum of digits at odd places $(A) = 2 + 7 + 8 = 17$

Sum of digits at even places $(B) = 1 + 3 + 2 = 6$

$$A - B = 17 - 6 = 11$$

Clearly, 217382 is divisible by 11.

Divisibility by 12 A number which is divisible by both 4 and 3 is also divisible by 12.

For example 2244 is divisible by both 3 and 4. Therefore, it is divisible by 12 also.

Divisibility by 14 A number which is divisible by both 7 and 2 is also divisible by 14.

For example 1232 is divisible by both 7 and 2. Therefore, it is divisible by 14 also.

Divisibility by 15 A number which is divisible by both 5 and 3 is divisible by 15 also.

For example 1275 is divisible by both 5 and 3. Therefore, it is divisible by 15 also.

Divisibility by 16 A number is divisible by 16 when the number made by its last 4-digits is divisible by 16.

For example 126304 is divisible by 16 as the number made by its last 4-digits *i.e.,* 6304 is divisible by 16.

Divisibility by 18 A number is divisible by 18 when it is even and divisible by 9. *For example* 936198 is divisible by 18 as it is even and divisible by 9.

Divisibility by 25 A number is divisible by 25 when its last 2-digits are either zero or divisible by 25.

For example 500, 1275, 13550 are divisible by 25 as last 2-digits of these numbers are either zero or divisible by 25.

Divisibility by 125 A number is divisible by 125 when the number made by its last 3-digits is divisible by 125.

For example 630125 is divisible by 125 as the number made by its last 3-digits are divisible by 125.

To Find a Number Completely Divisible by Given Number

Consider a number x, which is when divided by *d*, gives a quotient *q* and leaves a remainder *r*. Then,

$$d\overline{)\,x\,}(q \qquad \underline{\quad} \; r$$

To find the number which is completely divisible by *d* such that remainder *r* is zero, follows the example given below.

Ex. 1 Find the number, which on (1) addition (2) subtraction from the number 5029 is completely divisible by 17.

Sol. Dividing 5029 by 17 we find.

$\therefore$ Remainder = 14

1. The minimum number on adding of which the given number is completely divisible by 17 = Divisor − Remainder = 17 − 14 = 3.
2. The minimum number on subtraction of which the given number is completely divisible by 17 = Remainder = 14.

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17) 5029 (295
    34
    162
    153
     99
     85
     14
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Unit's Place of an Expression

Given expression can be of following two types

1. When Number is Given in the form of Product of Number

To find the units digit in the product of two or more number we take units digit of every numbers and then multiply them. Then, the unit digit of the resultant product is the units digit of the product of original numbers.

For example $207\times781\times39\times94$

Taking units digit of every number and then multiplying them

$=7\times1\times9\times4=7\times36$ [taking units place digit]

Again, taking units digit and then multiplying

$=7\times6=42$

$\therefore$ Units digit for $207\times781\times39\times94$ is 2.

2. When Number is Given in the form of Index

- If the unit's digit number are 0,1, 5 or 6, then the resultant unit's digit remains same.

  For example, $(576)^{1151}$, its units digit is 6. $(155)^{120}$, its unit digit is 5.

  $(191)^{19}$, its unit digit is 1. $(900)^{51}$, its unit digit is 0.

- If units place is 2, then the power of the number is first divided by 4 and there after represented in the form of 2^4.

For example $(572)^{443}$

i.e., $(2)^{443}=(2)^{\left(\frac{443}{4}\right)}$ taking units place digit

$=(2^4)^{110}\times2^3=2^4\times2^3$ $[\because (2^4)^{110}=2^4]$

$=16\times8$ taking units place digit

$\therefore$ Units place is 8

- In the same way, if units place digit are 4 or 8, then units digit for 4^4 and $8^4 = 6$.
  e.g.,
  $(124)^{372}$ taking units place digit $= (4)^{372} = (4^4)^{93} = 4^4 = 256$
  $\therefore$ Unit's place digit $= 6$
- If units digit is 3 or 7, then units digit for 3^4 and $7^4 = 1$.
  For example $(2467)^{153}$ taking units place $= (7)^{153} \Rightarrow (7^4)^{38} \times 7^1$
  $\Rightarrow$ $7^4 \times 7^1 = 7^2 \times 7^2 \times 7 = 49 \times 49 \times 7$ taking unit's place digit
  $= 9 \times 9 \times 7$
  $= 81 \times 7$ taking unit's place digit
  $= 7$
  $\therefore$ Unit's digit $= 7$
- If units place is 9 and if the power of 9 is even, then units digit will be 1 and if the power of 9 is odd, then units digit will be 9.
  For example $(539)^{140}$
  Since, power is even for unit's digit 9
  $\therefore$ Units digit $= 1$ $(539)^{141}$
  Since, power is odd for unit's digit 9
  $\therefore$ Units digit $= 9$

Basic Number Theory

- Square of every even number is an even number while square of every odd number is an odd number.
- A number obtained by squaring a number does not have 2, 3 ,7 or 8 at its unit place.
- Sum of first n natural numbers $= \dfrac{n(n+1)}{2}$
- Sum of first n odd numbers $= n^2$
- Sum of first n even numbers $= n(n+1)$
- Sum of square of first n natural numbers $= \dfrac{n(n+1)(2n+1)}{6}$
- Sum of cubes of first n natural numbers $= \left[\dfrac{n(n+1)}{2}\right]^2$
- There are 15 prime numbers between 1 and 50 and 10 prime numbers between 50 and 100.
- If p divides q and r, then p divides their sum and difference also.
  e.g., 4 divides 12 and 20, then $20 + 12 = 32$ and $20 - 12 = 8$ are also divisible by 4.
- For any natural number n, $(n^3 - n)$ is divisible by 6.
- The product of three consecutive natural numbers is always divisible by 6.
- $(x^m - a^m)$ is divisible by $(x - a)$ for all values of m.
- $(x^m - a^m)$ is divisible by $(x + a)$ for even values of m.
- $(x^m + a^m)$ is divisible by $(x + a)$ for odd values of m.
- Number of prime factors of $a^p\, b^q\, c^r\, d^s$ is $p + q + r + s$, where a, b, c and d are prime number.

Multi Concept QUESTIONS

1. *If n is any odd number greater than 1, then $n(n^2 - 1)$ is*

(a) divisible by 96 always
(b) divisible by 48 always
(c) divisible by 24 always
(d) None of these

➥ (*c*) Solving the question by taking two odd numbers greater than 1, *i.e.,* 3 and 5, then $n(n^2 - 1)$ for $n = 3$

$$\Rightarrow \quad 3(9-1) \Rightarrow 3 \times 8 = 24$$

$$n(n^2 - 1) \text{ for } n = 5 \Rightarrow 5(25-1) \Rightarrow 24 \times 5 = 120$$

Using option we find that both the number are divisible by 24.

2. $7^{6n} - 6^{6n}$, *where n is a integer greater than 0, is divisible by*

(a) 13
(b) 127
(c) 559
(d) None of these

➥ (*b*) $7^{6n} - 6^{6n}$ for $n = 1$, $7^6 - 6^6$

$$\Rightarrow \quad (7^3)^2 - (6^3)^2 \qquad \{a^2 - b^2 = (a+b)(a-b)\}$$

$$\Rightarrow \quad (7^3 - 6^3)(7^3 + 6^3) \Rightarrow (343 - 216)(343 + 216) \Rightarrow 127 \times 559$$

∴ It is clearly divisible by 127.

3. *Find the remainder of* $\dfrac{17^{18^{19^{20^{\cdots\infty}}}}}{8}$

(a) 2
(b) 3
(c) 4
(d) 1

➥ (*d*) $$\frac{17^{18^{19^{20^{\cdots\infty}}}}}{8} = \text{Remainder}\left\{\frac{(8 \times 2 + 1)^{18^{19^{20^{\cdots\infty}}}}}{8}\right\}$$

$$= \text{Remainder}\left\{\frac{(1)^{18^{19^{20^{\cdots\infty}}}}}{8}\right\} = \text{Remainder}\left\{\frac{1}{8}\right\}$$

∴ Remainder = 1

4. *If the sum of first 11 terms of an arithmetic progression equal that of the first 19 terms, Then, what is the sum of first 30 terms?*

(a) 0
(b) –1
(c) 1
(d) Not unique

➥ (*a*) Let the first term be *a* common difference of progression be *d*.
According to the question,

$$S_{11} = S_{19}$$

$$\frac{11}{2}[2a + 10d] = \frac{19}{2}[2a + 18d]$$

$$\Rightarrow \quad 16a + 232d = 0$$

$$\Rightarrow \quad 2a + 29d = 0$$

$$\therefore \quad S_{30} = \frac{30}{2}[2a + 29d] = \frac{30}{2} \times 0 = 0$$