Sets

In our mathematical language, everything in this universe—whether living or non-living—is called an object. Any collection of well-defined objects is called a set. By ‘well-defined objects’, we mean that given a set and an object, it must be possible to decide whether or not the object belongs to the set. The objects in set are called its members or elements.

Sets are denoted by capital letters A, B, C, etc., while the elements are denoted in general by small letters a, b, c, etc.

Following collections are sets
(i) The collection of all positive integers.
(ii) The collection of all capitals of states of India.

Let A be any set of objects and let ‘a’ be a member of A, then we write \( a \in A \) and read it as ‘a belongs to A’ or ‘a is an element of A’ or ‘a is member of A’. If \( a \) is not an object of A, then we write \( a \notin A \) and read as ‘a does not belong to A’ or ‘a is not an element of A’ or ‘a is not member of A’.

Some standard notations for some special points.
(i) The set of all natural numbers i.e., the set of all positive integers is denoted by \( N \).
(ii) The set of all integers is denoted by \( Z \) or \( I \).
(iii) The set of all rational numbers is denoted by \( Q \).
(iv) The set of all real numbers is denoted by \( R \).
(v) The set of all positive real numbers is denoted by \( R^+ \).
(vi) The set of all complex numbers is denoted by \( C \).
(vii) The set of all positive rational numbers is denoted by \( Q^+ \).

Types of Sets

(i) Empty set A set consisting of no element is called an empty set or null set or void set and is denoted by symbol \( \emptyset \) or \{ \}.
   \[ e.g., \emptyset = \{ x : x \in N \text{ and } 3 < x < 4 \} = \emptyset. \]
   A set which is not empty is called non-empty set or non-void set.
(ii) Singleton set A set consisting of only one element is called a singleton set.
   \[ e.g., \{2\}, \{0\}, \{\emptyset\} \]

(iii) Finite set A set having finite number of elements is called finite set.
   \[ e.g., A = \{1, 2, 3\} \text{ is a finite set.} \]

Representation of Sets

There are two methods to represent sets
(i) Listing method (ii) Set builder method
(iv) Infinite set  A set which is not finite is called an infinite set.

\[ \text{e.g., } A = \text{ Set of points lie in a plane is an infinite set.} \]

(v) Cardinal number of a finite set  The number of elements of a finite set \( A \) is called its cardinal number and it is denoted by \( n(A) \) or \( o(A) \).

(vi) Equivalent sets  Two finite sets \( A \) and \( B \) are said to be equivalent if they have the same cardinal number. Thus, sets \( A \) and \( B \) are equivalent if \( n(A) = n(B) \).

(vii) Subset and super set  The set \( B \) is said to be subset of set \( A \), if every element of set \( B \) is also an element of set \( A \). Symbolically we write it as, \( B \subseteq A \) or \( A \supseteq B \), where \( A \) is super set of \( B \).

\[ \begin{align*}
(a) & \quad B \subseteq A \text{ is read as } B \text{ is contained in } A \text{ or } B \text{ is subset of } A, \\
(b) & \quad A \supseteq B \text{ is read as } A \text{ contains } B \text{ or } B \text{ is a subset of } A.
\end{align*} \]

Evidently, if \( A \) and \( B \) are two sets such that \( x \in B \Rightarrow x \in A \), then \( B \) is subset of \( A \). The symbol ‘\( \Rightarrow \)’ stands for ‘implies’, we read it as \( x \) belongs to \( B \) implies that \( x \) belongs to \( A \).

\[ \text{e.g., Let } A = \{1, 2, 3, 4\}; B = \{1, 2, 4\} \]

Here, \( B \) is a subset of \( A \).

(viii) Proper subset  The set \( B \) is said to be a proper subset of set \( A \), if every element of set \( B \) is an element of \( A \) whereas every element of \( A \) is not an element of \( B \).

We write it as \( B \subset A \) and read it as \( B \) is a proper subset of \( A \). Thus, \( B \) is a proper subset of \( A \), if every element of \( B \) is an element of \( A \) and there is atleast one element in \( A \) which is not in \( B \).

Observe that \( A \subseteq A \) i.e., every set is a subset of itself, but not a proper subset.

\[ \text{e.g., Let } A = \{1, 2, 3\}; B = \{1, 2\}, \text{ then } B \subset A. \]

(ix) Equal sets  Two sets \( A \) and \( B \) are said to be equal, if each element of \( A \) is an element of \( B \) and each element of \( B \) is an element of \( A \).

Thus, two sets \( A \) and \( B \) are equal, if they have exactly the same elements but the order in which the elements in the two sets have been written down is immaterial.

\[ \text{Thus, if } x \in A \Rightarrow x \in B \text{ and } y \in B \Rightarrow y \in A, \text{ then } A \text{ and } B \text{ are equal} \]

\[ \text{e.g., } \{4, 8, 10\} = \{8, 4, 10\}, \text{ [The order in which the elements of a set is also immaterial]} \]

(x) Universal set  In any discussion in set theory, we need a set such that all sets under consideration in that discussion are its subsets. Such a set is called the universal set and is denoted by \( U \).

(xi) Power set  The set of all the subsets of a given set \( A \) is said to be the power set of \( A \) and is denoted by \( P(A) \).

\[ \text{e.g., If } A = \{1, 2, 3\}, \text{ then } P(A) = \{\emptyset, \{1\}, \{2\}, \{3\}, \{1, 2\}, \{2, 3\}, \{3, 1\}, \{1, 2, 3\}\}. \]

\[ \begin{align*}
\bullet & \quad \text{Elements of power set are the subset of } A. \\
\bullet & \quad \text{The power set of each given set is always non-empty.} \\
\bullet & \quad \text{If } A \text{ is a finite set of } n \text{ elements, then number of elements in } P(A) \text{ will be } 2^n. \\
\end{align*} \]

Venn Diagram

To express the relationship among sets in a perspective way, we represent them pictorially by means of diagrams, known as Venn diagrams.

\[ \text{The universal set is usually represented by a rectangular region and its subsets by circle or closed bounded regions inside this rectangular region.} \]

Operations on Sets

(i) Union of sets  Let \( A \) and \( B \) are two sets, then union of \( A \) and \( B \) is denoted by \( A \cup B \) and it consists of each one of which is either in \( A \) or in \( B \) or in both \( A \) and \( B \).

\[ \text{Thus, } A \cup B = \{x : x \in A \text{ or } x \in B\} \]

\[ \text{Clearly, } x \in A \cup B \Leftrightarrow x \in A \text{ or } x \in B \]

\[ \text{and } x \notin A \cup B \Leftrightarrow x \in A \text{ and } x \notin B \]

In the figure, the shaded part represents \( A \cup B \).

It is evident that \( A \subseteq A \cup B, B \subseteq A \cup B \).

Example 1. If \( A = \{1, 2, 3\} \) and \( B = \{1, 3, 5, 7\} \), then the value of \( A \cup B \) is

\[ \begin{align*}
(a) & \quad \{4, 5, 7\} \\
(b) & \quad \{1, 2, 3, 5, 7\} \\
(c) & \quad \{1, 2, 3, 5\} \\
(d) & \quad \{6, 3, 5, 7\}
\end{align*} \]

\[ \begin{align*}
\text{Solution} (b) & \quad A = \{1, 2, 3\} \text{ and } B = \{1, 3, 5, 7\} \\
\therefore & \quad A \cup B = \{1, 3, 5, 7\}
\end{align*} \]
(ii) **Intersection of sets** The intersection of two sets \(A\) and \(B\), denoted by \(A \cap B\) is the set of all elements, common to both \(A\) and \(B\).

Thus, \(A \cap B = \{x : x \in A \text{ and } x \in B\}\)

Clearly, \(x \in A \cap B \iff x \in A \text{ and } x \in B\)

In the figure, the shaded part represents \(A \cap B\).

It is evident that \(A \cap B \subseteq A\) and \(A \cap B \subseteq B\).

**Example 2.** If \(A = \{1, 2, 3, 4\}\) and \(B = \{2, 4, 6\}\), then the value of \(A \cap B\) is

(a) \(\{2, 4\}\) (b) \(\{6, 5\}\) (c) \(\{2, 3, 4\}\) (d) \(\{1, 2, 3\}\)

Solution (a) \(A \cap B = \{1, 2, 3, 4\}\) and \(B = \{2, 4, 6\}\)

\(\therefore\) \(A \cap B = \{2, 4\}\)

(iii) **Disjoint sets** Two sets \(A\) and \(B\) are said to be disjoint sets, if they have no common element i.e., \(A \cap B = \emptyset\).

The disjoint sets can be represented by Venn diagram as shown in the figure e.g., let \(A = \{1, 2, 3\}\) and \(B = \{4, 6\}\)

\(\therefore\) \(A \cap B = \emptyset\).

(iv) **Difference of sets** If \(A\) and \(B\) are two sets, then their difference \(A - B\) is the set of all those elements of \(A\) which do not belong to \(B\).

Thus, \(A - B = \{x : x \in A \text{ and } x \notin B\}\)

Clearly, \(x \in A - B \iff x \in A \text{ and } x \notin B\).

In the figure, the shaded part represents \(A - B\).

Similarly, the difference \(B - A\) is the set of all those elements of \(B\) that do not belong to \(A\) i.e.,

\(B - A = \{x : x \in B \text{ and } x \notin A\}\)

\(\therefore\) \(B - A = \{1, 9\}\) and \(B - A = \{2, 11\}\).

(v) **Symmetric difference of two sets** The symmetric difference of two sets \(A\) and \(B\), denoted by \(A \Delta B\) is the set \((A - B) \cup (B - A)\).

Thus, \(A \Delta B = (A - B) \cup (B - A) = \{x : x \notin A \cap B\}\)

The shaded part represents \(A \Delta B\).

**Example 3.** If \(A = \{1, 3, 5, 7, 9\}\) and \(B = \{2, 3, 5, 7, 11\}\), then find the value of \(A \Delta B\)

(a) \(\{5, 7, 11\}\) (b) \(\{1, 2, 9, 11\}\) (c) \(\{3, 5, 7\}\) (d) \(\{1, 3, 5, 11\}\)

Solution (b) \(A = \{1, 3, 5, 7, 9\}\) and \(B = \{2, 3, 5, 7, 11\}\)

\(\therefore\) \(A - B = \{1, 9\}\) and \(B - A = \{2, 11\}\)

\(\therefore\) \(A \Delta B = (A - B) \cup (B - A) = \{1, 9\} \cup \{2, 11\} = \{1, 2, 9, 11\}\)

**Complement of a Set**

If \(U\) is a universal set and \(A \subset U\), then complement set of \(A\) is denoted by \(A'\) or \(U - A\).

Thus, \(A' = U - A = \{x : x \in U, x \notin A\}\)

It is clear that \(x \in A' \iff x \notin A\)

\(\bullet\) \(\emptyset = U'\) \(\bullet\) \(\emptyset' = U\) \(\bullet\) \((A')' = A\)

\(\bullet\) \(A \cup A' = U\) \(\bullet\) \(A \cap A' = \emptyset\)

**Laws of Algebra of Sets**

If \(A, B\) and \(C\) are three sets, then

1. **Idempotent laws**
   (a) \(A \cup A = A\) \hspace{1cm} (b) \(A \cap A = A\)

2. **Identity laws**
   (a) \(A \cup \emptyset = A\) \hspace{1cm} (b) \(A \cap U = A\)

3. **Commutative laws**
   (a) \(A \cup B = B \cup A\) \hspace{1cm} (b) \(A \cap B = B \cap A\)

4. **Associative laws**
   (a) \((A \cup B) \cup C = (A \cup B) \cup C\) \hspace{1cm} (b) \((A \cap B) \cap C = (A \cap B) \cap C\)

5. **Distributive laws**
   (a) \((A \cup B) \cap C = (A \cup B) \cap C\) \hspace{1cm} (b) \((A \cap B) \cap C = (A \cap B) \cap C\)

6. **De-Morgan’s laws**
   (a) \((A \cup B)' = A' \cap B'\) \hspace{1cm} (b) \((A \cap B)' = A' \cup B'\)
7. (a) \( A - B = A \cap B' \)  
(b) \( B - A = B \cap A' \)  
(c) \( A - B = A \Rightarrow A \cap B = \phi \)  
(d) \((A-B) \cup B = A \cup B \)  
(e) \((A - B) \cap B = \phi \)  
(f) \((A - B) \cup (B - A) = (A \cup B) - (A \cap B) \)  

8. (a) \( A - (B \cap C) = (A - B) \cup (A - C) \)  
(b) \( A - (B \cap C) = (A - B) \cap (A - C) \)  
(c) \( A \cap (B - C) = (A \cap B) - (A \cap C) \)  
(d) \( A \cap (B \Delta C) = (A \cap B) \Delta (A \cap C) \)  

**Important Results**

If \( A, B \) and \( C \) are any three finite sets, then

1. \( n(A \cup B) = n(A) + n(B) - n(A \cap B) \)
2. \( n(A \cup B) = n(A) + n(B), \) if and only if \( A \cap B = \phi \)
3. \( n(A - B) = n(A) - n(A \cap B) \)
4. \( n(A \Delta B) = n(A - B) + n(B - A) = n(A) + n(B) - 2n(A \cap B) \)
5. \( n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(C \cap A) + n(A \cap B \cap C) \)
6. \( n(A^c \cup B^c) = n(U) - n(A \cap B) \)
7. \( n(A \cap B^c) = n(U) - n(A \cup B) \)

**Example 4.** If in a group of 850 persons, 600 can speak Hindi and 340 can speak Tamil. Then, the number of persons who can speak both Hindi and Tamil is

(a) \( 40 \)  
(b) \( 90 \)  
(c) \( 85 \)  
(d) \( 120 \)

**Solution**

(b) Let \( A \) and \( B \) denote the sets of persons who can speak Hindi and Tamil, respectively.

Then, \( n(A) = 600 \)  
\( n(B) = 340 \)  
and \( n(A \cup B) = 850 \)

Thus, 90 persons can speak both Hindi and Tamil.

**Ordered Pair**

Two elements \( a \) and \( b \) listed in a particular order, is called an ordered pair and it is denoted by \( (a, b) \). In an ordered pair \((a, b)\); ‘\( a \)’ is regarded as the first element and \( ‘b’ \) the second element.

It is evident from the definition that

(i) \( (a, b) \neq (b, a) \Rightarrow a \neq b \)

(ii) \( (a, b) = (c, d) \Rightarrow a = c, b = d \)

**Cartesian Product**

Let \( A \) and \( B \) be two non-empty sets. The cartesian product of \( A \) and \( B \), denoted by \( A \times B \), is defined as the set of all ordered pairs \((a, b)\), where \( a \in A \) and \( b \in B \).

Symbolically, \( A \times B = \{(a, b); \ a \in A \text{ and } b \in B \} \)

Thus, \( (a, b) \in A \times B \Leftrightarrow a \in A \text{ and } b \in B \)

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**Facts Related to Cartesian Product**

If \( A, B \) and \( C \) are non-empty sets, then

1. \( A \times (B \cup C) = (A \times B) \cup (A \times C) \)
2. \( A \times (B \cap C) = (A \times B) \cap (A \times C) \)
3. \( A \times B = B \times A \Leftrightarrow A = B \)
4. If \( A \subseteq B \Rightarrow A \times B \subseteq (A \times B) \cap (B \times A) \)
5. If \( A \subseteq B \Rightarrow A \times C \subseteq B \times C \)
6. If \( A \subseteq B \text{ and } C \subseteq D \Rightarrow A \times C \subseteq B \times D \)
7. \( (A \times B) \cap (C \times D) = (A \cap C) \times (B \cap D) \)
8. \( A \times (B^c \cup C^c) = (A \times B) \cap (A \times C) \)
9. \( A \times (B^c \cap C^c) = (A \times B) \cap (A \times C) \)
10. If \( n \) elements are common in \( A \) and \( B \), then in \( A \times B \) and \( B \times A \), \( n^2 \) elements will be common.

**Relation**

Let \( A, B \) and \( C \) be two non-empty sets, then a relation \( R \) from \( A \) to \( B \) is a subset of \( A \times B \).

Equivalently, any subset of \( A \times B \) is a relation from \( A \) to \( B \).

Thus, \( R \) is a relation from \( A \) to \( B \) if \( R \subseteq A \times B \)

\( \Leftrightarrow \ R \subseteq \{(a, b); a \in A, b \in B\} \)

If \((a, b) \in R\), then we write \( a \) \( R \) \( b \) which is read as ‘\( a \) is related to \( b \) by the relation \( R \).’ If \((a, b) \notin R\), then we write \( a R b \) and we say that \( a \) is not related to \( b \) by the relation \( R \).

**Domain and Range of Relation**

If \( R \) is a relation from set \( A \) to set \( B \), then the set of all first coordinates of elements of \( R \) is called the domain of \( R \), while the set of all second coordinates of elements of \( R \) is called the range of \( R \).

\[ \exists \text{ Domain (} R \text{)} = \{ a \in A; (a, b) \in R \text{ for some } b \in B \} \]

and \( \exists \text{ Range (} R \text{)} = \{ b \in B; (a, b) \in R \text{ for some } a \in A \} \)

**Codomain of a relation** If \( R \) be a relation from \( A \) to \( B \), then \( B \) is called the codomain of relation \( R \).

**Types of Relation**

(i) **Empty relation** Since, \( \phi \subseteq A \times A \), it follows that \( \phi \) is a relation on \( A \), called the empty or void relation.

\( e.g., \ L e t \ A = \{1, 2\}, \ B = \{1, 3\} \)

Let \( R = \{(1, 1), (1, 3), (2, 1), (2, 3)\} \)

Here, \( R = A \times B \)

Hence, \( R \) is the universal relation from \( A \) to \( B \).

(ii) **Universal relation** Since, \( A \times A \subseteq A \times A \), it follows that \( A \times A \) is a relation on \( A \), called the universal relation.

(iii) **Identity relation** The relation \( I_{A} = \{(a, a); a \in A\} \) is called the identity relation to \( A \).

\( e.g., \ L e t \ A = \{1, 2, 3\}, \text{ then the identity relation on } A \ \text{is given by} \ I_{A} = \{(1, 1), (2, 2), (3, 3)\} \).
(iv) **Inverse relation** If \( R \) is a relation on \( A \), then the relation \( R^{-1} \) on \( A \), defined by \( R^{-1} = \{ (b,a) : (a,b) \in R \} \) is called an inverse relation to \( R \).

Clearly, domain \( (R^{-1}) = \) range \( (R) \) and range \( (R^{-1}) = \) domain \( (R) \)

**Example 5.** Let \( A = \{1, 2, 3\} \) and let \( R = \{(1,2), (2,2), (3,3)\} \), then the domain and range of \( R \) is

(a) \( \{2, 1\} \) (b) \( \{1, 2\} \) (c) \( \{1, 2, 3\} \) (d) \( \{2, 3\} \)

**Solution**

(a) \( A = \{1, 2, 3\} \) and \( R = \{(1,1),(2,2)\} \)

Then, \( R \) being a subset of \( A \times A \), it is a relation on \( A \).

Clearly, \( R \) is reflexive, since 1 \( \in R \), 2 \( \in R \), and 3 \( \in R \).

(b) \( (1, 2) \) \( \in R \) and \( (2, 1) \) \( \in R \).

Thus, if \( x \) \( \in \) \( A \), then \( x \) is related to \( x \).

Clearly, \( \{1, 2, 3\} \) \( \subseteq \) \( \{1, 2, 3\} \) and \( \{1, 2\} \) \( \subseteq \) \( \{1, 2\} \).

(c) \( \{1, 2\} \) \( \subseteq \) \( \{1, 2\} \) and \( \{2, 3\} \) \( \subseteq \) \( \{2, 3\} \).

(d) \( \{2, 3\} \) \( \subseteq \) \( \{2, 3\} \) and \( \{1, 2\} \) \( \subseteq \) \( \{1, 2\} \).

**Various Types of Relations**

(i) **Reflexive relations** A relation \( R \) on a set \( A \) is said to be a reflexive relation, if \( (a,a) \in R \), \( \forall a \in A \). It should be noted, if \( \exists \) any \( a \in A \) such that \( (a,a) \notin R \), then \( R \) is not reflexive.

- \( e.g., \) let \( A = \{1, 2, 3\} \) and \( R = \{(1,1),(2,2)\} \)

Then, \( R \) is not reflexive, since 3 \( \notin \) \( A \) but \( (3,3) \notin R \).

(ii) **Symmetric relations** A relation \( R \) on a set \( A \) is said to be a symmetric relation on \( A \), if \( (x,y) \in R \Rightarrow (y,x) \in R \), \( \forall x,y \in R \). i.e., if \( R \) related to \( y \), then \( y \) is also \( R \) related to \( x \). It should be noted that \( R \) is symmetric, i.e. \( R^{-1} = R \).

- \( \text{Let} \ A = \{1, 2, 3\} \)

\( \text{Let} \ R_1 = \{(1,2),(2,1)\} \)

\( R_2 = \{(1,2),(2,1),(1,3),(3,1)\} \)

Here, \( R_1 \) and \( R_2 \) are symmetric relations on \( A \).

(iii) **Anti-symmetric relations** A relation \( R \) on a set \( A \) is said to be an anti-symmetric relation, if \( (a,b) \in R \) and \( (b,a) \in R \Rightarrow a = b \). Thus, if \( a \neq b \), then \( a \) may be related to \( b \) or \( b \) may be related to \( a \), but never both.

- \( e.g., \) let \( N \) be the set of natural numbers. A relation \( R \subseteq N \times N \) is defined by \( xRy \iff x \text{ divides } y \).

- \( \text{Let} \ A = \{1, 2, 3\} \)

\( R_1 = \{(1,2),(1,3),(1,1)\} \)

\( R_2 = \{(1,2)\} \)

Here, \( R_1 \) and \( R_2 \) are anti-symmetric relations on \( A \).

(iv) **Transitive relations** A relation \( R \) on a set \( A \) is said to be a transitive relation, if \( (a,b) \in R \), \( (b,c) \in R \Rightarrow (a,c) \in R \).

In other words, if \( a \) is related to \( b \), \( b \) is related to \( c \), then \( a \) is related to \( c \).

Transitivity fails only when there exists \( a,b \) and \( c \) such that \( aRb \), \( bRc \) but \( a Rc \).

- \( e.g., \) let \( A = \{1, 2, 3\} \) and the relation \( R = \{(1,2),(2,1),(1,1)\} \).

Then, \( R \) is not transitive, since \( (2,1) \in R \), \( (1,2) \in R \) but \( (2,2) \notin R \).

**Equivalence Relation**

A relation \( R \) on a set \( A \) is said to be an equivalence relation, if

(i) \( R \) is reflexive i.e., \( (a,a) \in R \), \( \forall a \in A \)

(ii) \( R \) is symmetric i.e., \( (a,b) \in R \Rightarrow (b,a) \in R \), \( \forall a,b \in A \)

(iii) \( R \) is transitive i.e., \( (a,b),(b,c) \in R \Rightarrow (a,c) \in R \)

**Example 6.** The relation ‘\( > \)’ on the set \( R \) of all real numbers is

(a) transitive
(b) reflexive
(c) symmetry
(d) anti-symmetric

**Solution** (a) The relation ‘\( > \)’ on \( R \) is transitive, since \( a > b \) and \( b > c \Rightarrow a > c \).

Since, no real number can be greater than itself, the relation is not reflexive.

Also, \( a > b \neq b > a \),

- \( e.g., \) \( 3 > 2 \) but \( 2 \neq 1 \).

So, it is not symmetric.

**Composition of Relations**

If \( R \) and \( S \) are relations from \( A \) to \( B \) and \( B \) to \( C \) respectively, then \( SoR \) is a relation from \( A \) to \( C \), which is defined as follows

\( (a,c) \in SoR \Leftrightarrow \exists b \text{ s.t. } (a, b) \in R \) and \( (b, c) \in S \).

This relation is known as composition of \( R \) and \( S \).

**Function**

Let \( A \) and \( B \) are two non-empty sets, then subset of \( A \times B \) i.e., \( f \) is known as a function from \( A \) to \( B \) if \( \forall a \in A \) there exists a unique element in \( B \) such that \( (a,b) \in f \).

Function is also known as mapping, transformations, operators.

Function is denoted by \( f : A \rightarrow B \) or \( A \longrightarrow B \).

**Domain, Codomain and Range of a Function**

Let \( f : A \rightarrow B \). Then, the set \( A \) is known as the domain of \( f \) and the set \( B \) is known as the codomain of \( f \). The set of all \( f \)-images of elements of \( A \) is known as the image of \( f \) or image set of \( A \) under \( f \) and is denoted by \( f(A) \).

Thus, \( f(A) = \{ f(x) : x \in A \} \) is Range of \( f \).

Clearly, \( f(A) \subseteq B \).
Various Types of Functions

(i) Many-one function  Let \( f : A \to B \). If two or more than two elements have the same image in \( B \), then \( f \) is said to be many-one function.

\[ f(x) = f(y) \Rightarrow x = y \quad \forall \ x, y \in A \]

\[ f(x) = f(y) \quad \text{then } f \text{ is said to be a constant function.} \]

Thus, \( f(x) = c \) for every \( x \in A \), where \( c \) is a fixed number Clearly, domain of \( f = A \) and range of \( f = \{ c \} \).

Thus, a function \( f : A \to B \) is a constant function, if range of \( f \) is a singleton set.

\[ \text{e.g.}, A = \{1, 2, 3\} \text{ and } B = \{5, 7\}, \text{ let } f : A \to B \text{ defined by } f(x) = 5, \forall x \in A. \]

Then, all the elements in \( A \) have the same image in \( B \). So, \( f \) is a constant function.

(ii) One-one function  (injective)  Let \( f : A \to B \). Then, \( f \) is said to be one-one function or an injective, if different elements of \( A \) have different images in \( B \).

Thus, \( f : A \to B \) is one-one iff for each \( a, b \in A \)

\[ a \neq b \Rightarrow f(a) \neq f(b), \quad \forall \ a, b \in A \]

\[ f(a) = f(b) \Rightarrow a = b, \quad \forall \ a, b \in A \]

\[ \text{e.g., the function } f : A \to B \text{ given by } f(x) = 2x \text{ is an one-one function.} \]

(iii) Onto function  (surjective)  Let \( f : A \to B \). If every element in \( B \) has atleast one preimage in \( A \), then \( f \) is said to be an onto function.

Thus, \( f : A \to B \) is a surjective, iff for each \( b \in B, \exists a \in A \) such that \( f(a) = b \) clearly, \( f \) is onto \( \Leftrightarrow \) range (\( f \)) = \( B \).

\[ f(x) = f(y) \Rightarrow x = y \quad \forall \ x, y \in A \]

\[ f(x) = f(y) \quad \Rightarrow \text{then } f \text{ is said to be an onto function.} \]

(iv) Into function  Let \( f : A \to B \). If there exists even a single element in \( B \) having no preimage in \( A \), then \( f \) is said to be an into function.

\[ f(x) = f(y) \Rightarrow \text{then } f \text{ is an into function.} \]

(v) Bijective function  A one-one and onto function is said to be bijective.

A bijective function is also known as a one-to-one correspondence.

In other words, a function \( f : A \to B \) is a bijection, if

(a) it is one-one i.e., \( f(x) = f(y) \Rightarrow x = y, \forall x, y \in A. \)

(b) it is onto i.e., \( \forall y \in B, \text{there exists } y \in A \text{ such that } f(x) = y. \)

(vi) Constant function  Let \( f : A \to B \) is defined in such a way that all the elements in \( A \) have the same image in \( B \), then \( f \) is said to be a constant function.

Thus, \( f(x) = c \) for every \( x \in A \), where \( c \) is a fixed number Clearly, domain of \( f = A \) and range of \( f = \{ c \} \).

Thus, a function \( f : A \to B \) is a constant function, if range of \( f \) is a singleton set.

\[ \text{e.g.}, A = \{1, 2, 3\} \text{ and } B = \{5, 7\}, \text{ let } f : A \to B \text{ defined by } f(x) = 5, \forall x \in A. \]

Then, all the elements in \( A \) have the same image in \( B \). So, \( f \) is a constant function.

(vii) Identity function  Let \( A \) be a non-empty set. Then, the function, defined by \( I_A : A \to A. I_A(x) = x, \forall x \in A, \) is called an identity function on \( A \).

This is clearly a one-one onto function with domain \( A \) and range \( A \).

(viii) Equal functions  Two functions \( f \) and \( g \) are said to be equal, written as \( f = g \), if they have the same domain and they satisfy the condition \( f(x) = g(x), \forall x \).

(ix) Even and odd functions  A function \( f : A \to B \) is said to be an even or odd function according as

\[ f(-x) = f(x), \quad \forall x \in A \quad \text{and} \quad f(-x) = -f(x), \quad \forall x \in A. \]

\[ f(x) = x^2 \]

\[ f(x) = x \quad \Rightarrow \text{then } f \text{ is an into function.} \]

Example 7.  Let \( N \) be the set of all natural numbers. If \( f : N \to N \) defined as \( f(x) = 2x, \forall x \in N \), then \( f \) is

(a) one-one onto \quad (b) many-one onto \quad (c) one-one into \quad (d) many-one into

Solution  (a) Clearly, each \( x \in N \) has its unique image \( 2x \in N \). So, \( f \) is onto.

Also, \( f \) is one-one, since

\[ f(x_1) = f(x_2) \Rightarrow 2x_1 = 2x_2 \Rightarrow x_1 = x_2 \]

Hence, \( f \) is one-one and onto.

(x) Inverse function  Let \( f \) be a one-one onto function from \( A \) to \( B \).

Let \( y \) be an arbitrary element of \( B \). Then, \( f \) being onto, there exists an element \( x \in A \), such that \( f(x) = y \).

Also, \( f \) being one-one, this \( x \) must be unique. Thus, for each \( y \in B \), there exists a unique element

\[ x \in A, \text{ such that } f(x) = y. \]

So, we may define a function, denoted by \( f^{-1} \) as

\[ f^{-1} : B \to A, \text{ such that } f^{-1}(y) = x \Rightarrow f(x) = y. \]

The above function \( f^{-1} \) is called the inverse of \( f \).
Example 8. The inverse of the function \( f : R \rightarrow R \) defined by 
\[
 f(x) = 4x - 7
\]
(a) \( \frac{x + 4}{2} \)
(b) \( \frac{x + 7}{4} \)
(c) \( \frac{x - 4}{5} \)
(d) does not exist

Solution (b) We have, \( f(x) = 4x - 7, x \in R \)
\[
f(x) = y \Rightarrow 4x - 7 = y
\]
\[
x = \frac{y + 7}{4}
\]
\[
\therefore f \text{ is onto.}
\]
To find \( f^{-1} \):
\[
f^{-1}(y) = \frac{y + 7}{4}
\]
\[
\therefore f^{-1} \text{ is one-one.}
\]

Composition of Functions
Let \( A, B \) and \( C \) be three non-empty sets. Let \( f : A \rightarrow B \) and \( g : B \rightarrow C \). Since, \( f : A \rightarrow B \), for each \( x \in A \), there exists a unique element \( g[f(x)] \) of \( C \). Thus, for each \( x \in A \), there is associated a unique element \( g[f(x)] \) of \( C \). Thus, from \( f \) and \( g \), we can define a new function from \( A \) to \( C \). This function is called the product or composite of \( f \) and \( g \), denoted by \( g \circ f \) and defined by
\[
(g \circ f) : A \rightarrow C \text{ such that } (g \circ f)(x) = g[f(x)] \text{ for all } x \in A.
\]

Properties of Composition of Functions
1. The product of any function with the identity function is the function itself.
2. The product of any invertible function \( f \) with its inverse function \( f^{-1} \) is an identity function.
3. Composite of functions is associative.
4. Let \( f : A \rightarrow B \) and \( g : B \rightarrow A \), such that \( g \circ f \) is an identity function on \( A \) and \( f \circ g \) is an identity function on \( B \). Then, \( g = f^{-1} \).
5. Let \( f : A \rightarrow B \) and \( g : B \rightarrow C \) be a one-one onto functions.

\[
\text{Then, } g \circ f \text{ is also one-one onto and } (g \circ f)^{-1} = f^{-1} \circ g^{-1}
\]

Comprehensive Approach
- A natural number \( p \) is a prime number, if \( p \) is greater than one and its factors are one and \('p'\) only.
- Finite sets are equivalent sets only when they have equal numbers of elements.
- Equal sets are equivalent sets, but equivalent sets may not be equal sets.
- Number of proper subsets of a set containing \( 'n' \) elements is \( 2^n - 1 \).
- If \( A \subseteq B \), we may have \( B \subseteq A \) but, if \( A \subset B \), we cannot have \( B \subset A \).
- \{x, y\} and \{y, x\} are equal sets but \((x, y)\) and \((y, x)\) are not equal ordered pairs.
- If \( 'A' \) is any set, then \( A \subseteq A \) is true but \( A \subset A \) is false.
- Total number of relations from set \( A \) to set \( B \) is equal to \( 2^{n(A) \times n(B)} \).
- The universal relation on a non-empty set is always reflexive, symmetric and transitive.
- The identity relation on a non-empty set is always reflexive, symmetric and transitive.
- The identity relation on a non-empty set is always anti-symmetric.
- If \( R \) is relation from \( A \) to \( B \) and \( S \) is a relation from \( B \) to \( C \), then \((R \circ S)^{-1} = S^{-1} \circ R^{-1}\).
- For two relations \( R \) and \( S \), the composite relation \( R \circ S \), \( S \circ R \) may be void relation.
- The product of two even or odd function is an even function.
- The product of an even and an odd function is an odd function.
- Every function \( f(x) \) can be expressed as the sum of an even and an odd function.
- If \( A \) and \( B \) have \( n \) and \( m \) distinct elements respectively, then the number of mappings from \( A \) to \( B \) is equal to \( m^n \).
- If \( A \) and \( B \) have \( n \) distinct elements, then the number of mappings from \( A \) to \( B \) is equal to \( n^m \).
- The number of one-one functions that can be defined from a finite set \( A \) into a finite set \( B \) is \( n(B) \times \frac{P(n(A), A)}{n(A)} \) if \( n(B) \geq n(A) \) and zero, otherwise.
- The number of onto functions that can be defined from a finite set \( A \) containing \( n \) elements on finite set \( B \) containing \( 2 \) elements is \( 2^n - 2 \).
- If two curves do not intersect each other, then intersection of two sets is a null set.
Exercise

Level I

1. If \( A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3)\} \), then \( A \) is equal to
   (a) \{1, 2\}  (b) \{1, 2, 3\}  (c) \{2, 3\}  (d) None of these

2. If \( A = \{1, 2, 3\} \) and \( B = \{3, 4\} \), then \((A \cup B) \times (A \cap B)\) is
   (a) \{3, 3\}  (b) \{(1, 3), (2, 3), (3, 3), (1, 4), (2, 4), (3, 4)\}  (c) \{(1, 3), (2, 3), (3, 3)\}  (d) \{(1, 3), (2, 3), (3, 3), (4, 3)\}

3. Which of the following is correct?
   (a) \( A \cap B \subseteq A \cup B \)  (b) \( A \subseteq A \cap B \)  (c) \( A \cup B \subseteq A \cap B \)  (d) None of these

4. If \( n(A) = 8 \), \( n(A \cap B) = 2 \), then \( n(A - B) \) is equal to
   (a) 8  (b) 2  (c) 6  (d) 9

5. A set contains \( n \) elements. Then, the power set contains
   (a) \( n \) elements  (b) \( n \) elements  (c) \((2^n - 1)\) elements  (d) \( 2^n \) elements

6. If \( A = \{1, 2, 3, 4\} \) and \( B = \{5, 6, 7\} \), then number of relations from \( A \) to \( B \) is equal to
   (a) \( 2^4 \)  (b) \( 2^6 \)  (c) \( 2^7 \)  (d) \( 2^{12} \)

7. If \( \phi \) is a null set, then which one of the following is correct?
   (a) \( \phi = \emptyset \)  (b) \( \phi = \{0\} \)  (c) \( \phi = \{\} \)  (d) \( \phi = \{1\} \)

8. If \( A = \{a, b, c\} \), then what is the number of proper subsets of \( A \)?
   (a) 5  (b) 6  (c) 7  (d) 8

9. If \( A = \{1, 2, 5, 6\} \) and \( B = \{1, 2, 3\} \), then what is \((A \times B) \cap (B \times A)\) equal to?
   (a) \{1, (1, 2), (6, 1), (3, 2)\}  (b) \{(1, 1), (1, 2), (2, 1), (2, 2)\}  (c) \{(1, 1), (2, 2)\}  (d) \{(1, 1), (1, 2), (2, 5), (2, 6)\}

10. If \( E \) is the universal set and \( A = B \cup C \), then the set \((E - (E - (E - (E - A)))\) is the same as the set
    (a) \( B^c \cup C^c \)  (b) \( B \cup C \)  (c) \( B^c \cap C^c \)  (d) \( B \cap C \)

11. If a set \( A \) contains 3 elements and another set \( B \) contains 6 elements, then the number of elements in \( A \cup B \) would be
    (a) 9  (b) either 8 or 9  (c) either 7 or 8 or 9  (d) either 6 or 7 or 8 or 9

12. In a class of 100 students, 70 have taken Science, 60 have taken Mathematics, 40 have taken both Science and Mathematics. The number of students who have not taken Science or Mathematics or both Science and Mathematics, is equal to
    (a) 90  (b) 10  (c) 30  (d) 20

13. The relation \( R \) on a set \( A = \{1, 2, 3, 4\} \) is defined as \( \{(1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 2)\} \). Then, \( R \) is
    (a) reflexive  (b) symmetric  (c) anti-symmetric  (d) transitive

14. If \( A \), \( B \) and \( C \) are non-empty sets such that \( A \cap C = \phi \), then what is \((A \times B) \cap (C \times B)\) equal to? (NDA 2011 I)
    (a) \( A \times C \)  (b) \( A \times B \)  (c) \( B \times C \)  (d) \( \phi \)

15. If \( A = \{4n + 2\} \) is a natural number and \( B = \{3n\} \) is a natural number, then what is \((A \times B)\) equal to? (NDA 2011 I)
    (a) \( \{12n^2 + 6n\} \) is a natural number
    (b) \( \{24n - 12\} \) is a natural number
    (c) \( \{12n^2 - 6n\} \) is a natural number
    (d) \( \{12n - 6\} \) is a natural number

16. If \( A \) and \( B \) are two disjoint sets, then which one of the following is correct? (NDA 2010 II)
    (a) \( A - B = A - (A \cap B) \)  (b) \( B - A' = B \cap A \)  (c) \( A \cap B = (A \cap B) \cap B \)  (d) All of these

17. If the cardinality of a set \( A \) is 4 and that of a set \( B \) is 3, then what is the cardinality of the set \( A \Delta B \)?
    (a) 1  (b) 5  (c) 7  (d) Cannot be determined as the sets \( A \) and \( B \) are not given.

18. If \( A \) and \( B \) are finite sets, then which one of the following is the correct equation?
    (a) \( n(A - B) = n(A) - n(B) \)  (b) \( n(A - B) = n(B - A) \)  (c) \( n(A - B) = n(A) - n(A \cap B) \)  (d) \( n(A - B) = n(B - A) - n(A \cap B) \)
    [\( n(A) \) denotes the number of elements in \( A \)]

19. Which one of the following is correct?
    The relation \( R = \{(1, 1), (2, 2), (3, 3)\} \) on a set \( A = \{1, 2, 3\} \) is
    (a) only reflexive  (b) only symmetric  (c) only transitive  (d) reflexive, symmetric and transitive

20. If \( X = \{\text{multiples of 2}\} \), \( Y = \{\text{multiples of 5}\} \), \( Z = \{\text{multiples of 10}\} \), then \( X \cap (Y \cap Z) \) is equal to
    (a) \{multiples of 10\}  (b) \{multiples of 5\}  (c) \{multiples of 2\}  (d) \{multiples of 20\}
21. Let $X$ be any non-empty set containing $n$ elements. Then, what is the number of relations on $X$?
(a) $2^{n^2}$ (b) $2^n$ (c) $2^{2n}$ (d) $n^2$

22. Which of the following is a null set?
(a) $\{x : |x| < 1, x \in N\}$
(b) $\{x : |x| = 5, x \in N\}$
(c) $\{x : x^2 = 1, x \in Z\}$
(d) $\{x : x^2 + 2x + 1 = 0, x \in R\}$

23. The set of intelligent students in a class is
(a) a null set
(b) a singleton set
(c) a finite set
(d) not a well defined collection

24. If $A$ is the number of relations from $A$ to $B$, how many will $A \times B$ have in common? Which one of the following is correct?
(a) $2$ (b) $3$ (c) $5$ (d) $9$

25. If $A = P\{1, 2\}$, where $P$ denotes the power set, then which one of the following is correct?
(a) $\{1, 2\} \subset A$ (b) $1 \in A$ (c) $\phi \notin A$ (d) $\{1, 2\} \in A$

26. If $A$ and $B$ are any two sets, then what is the value of $A \cap (A \cup B)$?
(a) Complement of $A$ (b) Complement of $B$ (c) $B$ (d) $A$

27. Let $A = \{x : x$ is a digit in the number 3591\}$ $B = \{x : x \in N, x < 10\}$, which of the following is not correct?
(a) $A \cap B = \{1, 3, 5, 9\}$
(b) $A - B = \phi$
(c) $B - A = \{2, 4, 6, 7, 8\}$
(d) $A \cup B = \{1, 2, 3, 5, 9\}$

28. If $A = \{(x, y) : y = e^x, x \in R\}$ and $B = \{(x, y) : y = e^{-x}, x \in R\}$, then $A \cap B$ is
(a) empty set (b) singleton set (c) not a set (d) None of these

29. The relation $R$ defined on set $A = \{x : |x| < 3, x \in Z\}$ by $R = \{(x, y) : y = |x|\}$ is
(a) $\{(-2, 2), (-1, 1), (0, 0), (1, 1), (2, 2)\}$
(b) $\{(-2, -2), (-1, -1), (0, 0), (1, 1), (2, 2)\}$
(c) $\{(0, 0), (1, 1), (2, 2)\}$
(d) None of the above

30. Let $A = \{2, 3, 4, 5\}$ and $R = \{(2, 2), (3, 3), (4, 4), (5, 5)\}$ be a relation in $A$. Then, $R$ is
(a) reflexive (b) symmetric (c) transitive (d) None of these

31. For non-empty subsets $A$, $B$ and $C$ of a set $X$ such that $A \cup B = B \cap C$, which one of the following is the strongest inference that can be derived?
(a) $A = B = C$ (b) $A \subseteq B = C$ (c) $A = B \subseteq C$ (d) $A \subseteq B \subseteq C$

32. The function $f(x) = \frac{x}{x^2 + 1}$ from $R$ to $R$ is
(a) one-one as well as onto (b) onto but not one-one (c) neither one-one nor onto (d) one-one but not onto

33. Let $A = \{-1, 2, 5, 8\}$, $B = \{0, 1, 2, 6, 7\}$ and $R$ be the relation 'is one less than' from $A$ to $B$, then how many elements will $R$ contain?
(a) 2 (b) 3 (c) 5 (d) 9

34. If $n(A) = 115$, $n(B) = 326$, $n(A - B) = 47$, then what is $n(A \cup B)$ equal to?
(a) 373 (b) 165 (c) 370 (d) 394

35. If $P(A)$ denotes the power set of $A$ and $A$ is the void set, then what is number of elements in $P(P(P(P(A))))$?
(a) 0 (b) 1 (c) 4 (d) 16

36. If $N_a = \{ax | x \in N\}$, then what is $N_{12} \cap N_a$ equal to?
(a) $N_{12}$ (b) $N_{20}$ (c) $N_{24}$ (d) $N_{48}$

37. Consider the following Venn diagram

If $|E| = 42$, $|A| = 15$, $|B| = 12$ and $|A \cup B| = 22$, then the area represented by the shaded portion in the above Venn diagram is
(a) 25 (b) 27 (c) 32 (d) 37

38. If $A$, $B$ and $C$ are three sets and $U$ is the universal set such that $n(U) = 700$, $n(A) = 200$, $n(B) = 300$ and $n(A \cap B) = 100$, then what is the value of $(A' \cap B')$?
(a) 100 (b) 200 (c) 300 (d) 400

39. Let $A = \{x : x$ is a square of a natural number and $x$ is less than 100\}$ and $B$ is a set of even natural numbers. What is the cardinality of $A \cap B$?
(a) 4 (b) 5 (c) 9 (d) None of these

40. If $X = \{(4^n - 3n - 1) | n \in N\}$ and $Y = \{9(n-1) | n \in N\}$, then $X \cup Y$ equals to
(a) $X$ (b) $Y$ (c) $N$ (d) a null set

41. Sets $A$ and $B$ have $n$ elements in common. How many elements will $(A \times B)$ and $(B \times A)$ have in common?
(a) 0 (b) 1 (c) $n$ (d) $n^2$
42. What is the number of proper subsets of a given finite set with \( n \) elements?

(a) \( 2n - 1 \)  
(b) \( 2n - 2 \)  
(c) \( 2^n - 1 \)  
(d) \( 2^n - 2 \)  

43. If \( A, B \) and \( C \) are three sets, such that \( A \cup B = A \cup C \) and \( A \cap B = A \cap C \), then which one of the following is correct?

(a) \( A = B \) only  
(b) \( B = C \) only  
(c) \( A = C \) only  
(d) \( A = B = C \)  

44. If \( A, B \) and \( C \) are three sets, then \( A - (B - C) \) equals to

(a) \( A - (B \cap C) \)  
(b) \( (A - B) \cup C \)  
(c) \( (A - B) \cup (A \cap C) \)  
(d) \( (A - B) \cup (A - C) \)  

45. In a class containing 120 students, 65 students drink tea and 84 students drink coffee. If \( x \) students drink both tea and coffee, then what is the value of \( x \)?

(a) \( 39 \)  
(b) \( 65 \)  
(c) \( 29 \leq x \leq 65 \)  
(d) \( 29 \leq x \leq 84 \)  

46. If \( A \) and \( B \) are two sets satisfying \( A - B = B - A \), then which one of the following is correct?

(a) \( A = \emptyset \)  
(b) \( A \cap B = \emptyset \)  
(c) \( A = B \)  
(d) None of these  

47. Two finite sets have \( m \) and \( n \) elements, respectively. The total number of subsets of the first set is 56 more than the total number of subsets of the second set. What are the values of \( m \) and \( n \), respectively?

(a) \( 7, 6 \)  
(b) \( 6, 3 \)  
(c) \( 5, 1 \)  
(d) None of these

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**Level II**

1. Consider the following in respect of subsets \( A \) and \( B \) of \( X \):

I. \( A \subseteq B \Longleftrightarrow A \cup B = A \)  
II. \( (A \cup B) \cap (B \cup A) = A \cap B \)  
III. \( A \cap B = B \cap A \)  
IV. \( A \cap B = A \cap (B \cup A) \)  

Which of these are correct?

(a) I and III  
(b) I and IV  
(c) II and III  
(d) None of these

2. Consider the following statements:

I. All poets (\( P \)) are learned people (\( L \)).  
II. All learned people (\( L \)) are happy people (\( H \)).

Which one of the following Venn diagrams correctly represents both the above statements taken together?

(a) 
(b) 
(c) 
(d)

3. If \( R \) be a relation on \( N \times N \) defined by \( (a, b) R (c, d) \) if and only if \( ad = bc \); then \( R \) is

(a) an equivalence relation  
(b) symmetric and transitive but not reflexive  
(c) reflexive and transitive but not symmetric  
(d) reflexive and symmetric but not transitive

4. If \( A = \{1, 2, 3, 4\} \) and \( B = \{2, 3, 5\} \), then identify the correct relation, among the following from \( A \) to \( B \) given by \( xRy \), if and only if \( x < y \)

(a) \( R = \{(1, 2), (1, 3), (2, 2), (2, 3)\} \)  
(b) \( R = \{(3, 2), (3, 3), (3, 4), (3, 5)\} \)  
(c) \( R = \{(1, 2), (1, 3), (2, 3), (2, 5)\} \)  
(d) \( R = \{(1, 3), (1, 5), (3, 2), (4, 2)\} \)

5. What is the range of \( f(x) = \cos 2x - \sin 2x \)?

(a) \([2, 4]\)  
(b) \([-\sqrt{2}, \sqrt{2}]\)  
(c) \([-\sqrt{2}, \sqrt{2}]\)  
(d) \([-\sqrt{2}, \sqrt{2}]\)

6. Which one of the following is the union of the closed sets \( [2 + \frac{1}{n}, 10 - \frac{1}{n}] \), \( n = 1, 2, ... \)?

(a) \([2, 10]\)  
(b) \((2, 10)\)  
(c) \([2, 10]\)  
(d) \((2, 10)\)

7. \( Q = (\sin \theta + \cos \theta) \). Which one of the following is the correct range of \( Q \)?

(a) \(-2 \leq Q < 2\)  
(b) \(-2 < Q < 2\)  
(c) \(-\sqrt{2} \leq Q \leq \sqrt{2}\)  
(d) None of these

8. What does the shaded portion of the Venn diagram given above represent?

(a) \((P \cap Q) \cap (P \cap R)\)  
(b) \((P \cap Q) \cup (P \cap R) \cap Q\)  
(c) \((P \cup Q) \cap R \cap (P \cup Q) \cap R\) \((P \cup Q) \cap R\)  
(d) \((P \cap Q) \cap R \cap (P \cup Q) \cap R\)

9. Consider the following with regard to a relation \( R \) on a set of real numbers defined by \( xRy \) if and only if \( 3x + 4y = 5 \)

I. \( 0R1 \)  
II. \( 1R\frac{1}{2} \)  
III. \( \frac{2}{3} R \frac{3}{4} \)

Which of the above are correct?

(a) I and II  
(b) I and III  
(c) II and III  
(d) I, II and III
11. Let $N$ denote the set of natural numbers and $A = \{n^2 : n \in N\}$ and $B = \{n^{3/2} : n \in N\}$. Which one of the following is correct? (NDA 2010 II)

(a) $A \cup B = N$
(b) The complement of $(A \cup B)$ is a finite set
(c) $A \cap B$ must be a finite set
(d) $A \cap B$ must be a proper subset of $\{m^6 : m \in N\}$

12. The relation $R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\}$ on a set $A = \{1, 2, 3\}$ is (NDA 2010 II)

(a) reflexive, transitive but not symmetric
(b) reflexive, symmetric but not transitive
(c) symmetric, transitive but not reflexive
(d) reflexive but neither symmetric nor transitive

13. Let $R$ be a set of real numbers and let $S$ be a relation defined on $R$ as follows

$xSy$ if and only if, $x^2 + y^2 = 1$

Which one of the following statements is correct?

(a) $S$ is a reflexive relation
(b) $S$ is a symmetric relation
(c) $S$ is a transitive relation
(d) $S$ is an anti-symmetric relation

14. If $F(n)$ denotes the set of all divisors of a natural number $n$, what is the least value of $y$ satisfying $|F(20) \cap F(16)| = F(y)$?

(a) 2 (b) 1 (c) 4 (d) 6

15. The Venn diagram shown above represents four sets of people who can speak Telugu ($T$), English ($E$), Hindi ($H$) and Kannada ($K$). What does the marked region represent?

(a) People, who can speak Hindi and Kannada only
(b) People, who can speak English, Telugu and Kannada only
(c) People, who can speak Hindi and English only
(d) People, who can speak Hindi, English and Kannada only

16. The function $f(x) = e^x, x \in R$ is (NDA 2010 II)

(a) onto but not one-one
(b) one-one onto
(c) one-one but not onto
(d) neither one-one nor onto

17. Consider the function $f : R \rightarrow \{0, 1\}$ such that

$$f(x) = \begin{cases} 
1, & \text{if } x \text{ is rational} \\
0, & \text{if } x \text{ is irrational}
\end{cases}$$

Which one of the following is correct? (NDA 2010 II)

(a) The function is one-one into
(b) The function is many-one into
(c) The function is one-one onto
(d) The function is many-one onto

18. Which one of the following functions $f : R \rightarrow R$ is injective? (NDA 2009 II)

(a) $f(x) = |x|, \forall x \in R$
(b) $f(x) = x^2, \forall x \in R$
(c) $f(x) = 11, \forall x \in R$
(d) $f(x) = -x, \forall x \in R$

19. If $A = \{a, b, c\}$ and $R = \{(a, a), (a, b), (b, c), (b, b), (c, c), (c, a)\}$ is a binary relation on $A$, then which one of the following is correct? (NDA 2009 I)

(a) $R$ is reflexive and symmetric but not transitive
(b) $R$ is reflexive and transitive but not symmetric
(c) $R$ is reflexive but neither symmetric nor transitive
(d) $R$ is reflexive symmetric and transitive

20. If $\alpha, \beta, \xi$ and $\eta$ are non-empty sets, then

(a) $(\alpha \times \beta) \cup (\xi \times \eta) = (\alpha \times \beta) \cap (\xi \times \eta)$
(b) $(\alpha \times \beta) \cap (\xi \times \eta) = (\alpha \times \beta) \cap (\eta \times \xi)$
(c) $(\alpha \cap \beta) \times (\xi \cap \eta) = (\alpha \times \xi) \cup (\beta \times \eta)$
(d) $(\alpha \cap \beta) \times (\xi \cap \eta) = (\alpha \times \eta) \cap (\beta \times \xi)$

21. In a Euclidean plane, which one of the following is not an equivalence relation?

(a) Parallelism of lines (a line being deemed parallel to itself)
(b) Congruence of triangles
(c) Similarity of triangles
(d) Orthogonality of lines

22. Which one of the following is correct?

(a) The relation $R_0$ defined on the set of real numbers as $R_0 = \{(a, b) \mid a^2 + b^2 = 1\}$ for all $a, b \in R$ is an equivalence relation
(b) The relation $R_0$ defined on the set of real numbers as $R_0 = \{(a, b) \mid |a - b| \leq \frac{1}{3}\}$ for all $a, b \in R$ is an equivalence relation
(c) The relation $I_0$ defined on the set of integers as $I_0 : I_1I_2 = I_1 \cap I_2 - 3I_1I_2 + 2I_1 = 0$ for all $I_1, I_2 \in I$ is an equivalence relation
(d) We define $AE.B$ by the open sentence : $A$ is cardinally equivalent to $B$ on the family of sets Then, the relation $E_c$ on family of sets is an equivalence relation

23. Let $g : R \rightarrow R$ be a function such that, $g(x) = 2x + 5$. Then, what is the value of $g^{-1}(x)$? (NDA 2008 II)

(a) $\frac{x - 5}{2}$  (b) $2x - 5$
(c) $\frac{x - 5}{2}$  (d) $\frac{x + 5}{2}$
24. If \( A = \{1, 2, 3, 4\} \) and \( R = \{(1, 1), (1, 3), (2, 2), (3, 1), (3, 4), (4, 3), (4, 4)\} \) is a relation on \( A \times A \), then which one of the following is correct? (NDA 2008 II)
   (a) \( R \) is reflexive
   (b) \( R \) is symmetric and transitive
   (c) \( R \) is transitive, but not reflexive
   (d) \( R \) is neither reflexive nor transitive

25. The function \( f : R \to R \) defined by \( f(x) = (x^2 + 1)^{35} \) for all \( x \in R \) is (NDA 2008 II)
   (a) one-one but not onto
   (b) onto but not one-one
   (c) Neither one-one nor onto
   (d) Both one-one and onto

26. Let \( N \) be the set of integers. A relation \( R \) on \( N \) is defined as \( R = \{(x, y) \mid xy > 0, x, y \in N\} \). Then, which one of the following is correct? (NDA 2007 II)
   (a) \( R \) is symmetric but not reflexive
   (b) \( R \) is reflexive but not symmetric
   (c) \( R \) is symmetric and reflexive but not transitive
   (d) \( R \) is an equivalence relation

27. Let \( R \) be the relation defined on the set of natural number \( N \) as \( aRb; \ a, b \in N \), if \( a \) divides \( b \). Then, which one of the following is correct? (NDA 2008 I)
   (a) \( R \) is reflexive only
   (b) \( R \) is symmetric only
   (c) \( R \) is transitive only
   (d) \( R \) is reflexive and transitive

28. Consider the following statements (NDA 2008 I)
   I. \( \emptyset \subseteq \{\emptyset\} \)
   II. \( \{\emptyset\} \subseteq \emptyset \)
   Which of the statements given above is/are correct?
   (a) I only
   (b) II only
   (c) Both I and II
   (d) Neither I nor II

29. Which one of the following is correct? (NDA 2008 I)
   (a) \( A \cup P(A) = P(A) \)
   (b) \( A \cap P(A) = A \)
   (c) \( A \subset P(A) = A \)
   (d) \( P(A) - \{A\} = P(A) \)
   Here, \( P(A) \) denotes the power set of a set \( A \).

30. If \( A, B \) and \( C \) are three finite sets, then \( |(A \cup B) \cap C| \) equals to (NDA 2009 I)
   (a) \( A' \cup B' \cap C' \)
   (b) \( A' \cap B' \cap C' \)
   (c) \( A' \cap B' \cup C' \)
   (d) \( A \cap B \cap C \)

31. If \( A \) and \( B \) are two non-empty sets having \( n \) elements in common, then what is the number of common elements in the sets \( A \times B \) and \( B \times A \)? (NDA 2012 I)
   (a) \( n \)
   (b) \( n^2 \)
   (c) \( 2n \)
   (d) Zero

32. If \( A = \{x: x^2 = 1\} \) and \( B = \{x: x^4 = 1\} \), then \( A \Delta B \) is equal to (NDA 2008 II)
   (a) \( \{i, -i\} \)
   (b) \( \{-1, 1\} \)
   (c) \( \{-1, 1, -i, i\} \)
   (d) None of these

33. There are 100 families in a society, 40 families buy newspaper \( A \), 30 families buy newspaper \( B \), 30 families buy newspaper \( C \), 10 families buy newspapers \( A \) and \( B \), 8 families buy newspapers \( B \) and \( C \), 5 families buy newspaper \( A \) and \( C \), 3 families buy newspapers \( A \), \( B \) and \( C \), then the number of families, who do not buy any newspaper is
   (a) 20
   (b) 80
   (c) 0
   (d) None of these

34. Two finite sets \( A \) and \( B \) having \( m \) and \( n \) elements. The total number of relation from \( A \) to \( B \) is 64, then possible values of \( m \) and \( n \) are
   (a) 2 and 4
   (b) 2 and 3
   (c) 2 and 1
   (d) 64 and 1

35. Suppose \( A_1, A_2, \ldots, A_{30} \) are thirty sets each having 5 elements and \( B_1, B_2, \ldots, B_n \) are \( n \) sets each having 30 elements. Let \( \cup_{i=1}^{30} A_i = \cup_{j=1}^{n} B_j = S \) and each elements of \( S \) belongs to exactly 10 of \( A_i \)'s and exactly 9 of \( B_j \)'s. The value of \( n \) is equal to
   (a) 15
   (b) 3
   (c) 45
   (d) None of these

36. Universal set,
   \( U = \{x: x^2 - 6x^2 + 11x^2 - 6x^2 = 0\} \)
   \( A = \{x: x^2 - 5x + 6 = 0\} \)
   \( B = \{x: x^2 - 3x + 2 = 0\} \)
   What is the value of \( (A \cap B)' \)?
   (a) \{1, 3\}
   (b) \{1, 2, 3\}
   (c) \{0, 1, 3\}
   (d) \{0, 1, 2, 3\}

37. A relation \( R \) is defined on the set \( Z \) of integers as follows \( mRn \Leftrightarrow m + n \) is odd.
   Which of the following statements is/are true for \( R \)?
   I. \( R \) is reflexive. II. \( R \) is symmetric. III. \( R \) is transitive.
   Select the correct answer using the code given below
   (a) II only
   (b) II and III
   (c) I and II
   (d) I and III

38. Which of the following statements is not correct for the relation \( R \) defined by \( aRb \) if and only if \( b \) lives within 1 km from \( a \)?
   (a) \( R \) is reflexive
   (b) \( R \) is symmetric
   (c) \( R \) is not anti-symmetric
   (d) None of the above

39. What is the region that represents \( A \cap B \), if
   \( A = \{(x, y) | x + y \leq 4\} \) and \( B = \{(x, y) | x + y \leq 0\} \)?
   (a) \( \{(x, y) | x + y \leq 2\} \)
   (b) \( \{(x, y) | 2x + y \leq 4\} \)
   (c) \( \{(x, y) | x + y \leq 0\} \)
   (d) \( \{(x, y) | x + y \leq 4\} \)

40. Let \( X \) and \( Y \) be two non-empty sets and let \( R_1 \) and \( R_2 \) be two relations from \( X \) into \( Y \). Then, which one of the following is correct?
   (a) \( (R_1 \cap R_2)^{-1} \subset R_1^{-1} \cap R_2^{-1} \)
   (b) \( (R_1 \cap R_2)^{-1} \supset R_1^{-1} \cap R_2^{-1} \)
   (c) \( (R_1 \cap R_2)^{-1} = R_1^{-1} \cap R_2^{-1} \)
   (d) \( (R_1 \cap R_2)^{-1} = R_1^{-1} \cup R_2^{-1} \)

41. If a set \( A \) contains 4 elements, then what is the number of elements in \( A \times P(A) \)? (NDA 2008 II)
   (a) 16
   (b) 32
   (c) 64
   (d) 128
42. Let \( f : R \rightarrow R \) be a function defined as \( f(x) = x \cdot x \); for each \( x \in R \), \( R \) being the set of real numbers. Which one of the following is correct? \( \text{(NDA 2009 I)} \)

(a) \( f \) is one-one but not onto
(b) \( f \) is onto but not one-one
(c) \( f \) is both one-one and onto
(d) \( f \) is neither one-one nor onto

43. If \( A \) and \( B \) are subsets of a set \( X \), then what is the value of \( |A \cap (X - B)| \cup B? \) \( \text{(NDA 2009 I)} \)

(a) \( A \cup B \)  
(b) \( A \cap B \)  
(c) \( A \)  
(d) \( B \)

44. Let \( U = \{x \in N : 1 \leq x \leq 10\} \) be the universal set, \( N \) being the set of natural numbers. If \( A = \{1, 2, 3, 4\} \) and \( B = \{2, 3, 6, 10\} \), then what is the complement of \( A - B? \) \( \text{(NDA 2012 I)} \)

(a) \( \{6, 10\} \)  
(b) \( \{1, 4\} \)  
(c) \( \{2, 3, 5, 6, 7, 8, 9, 10\} \)  
(d) \( \{5, 6, 7, 8, 9, 10\} \)

Directions (Q. Nos. 45-47) Each of these questions contain two statements, one is Assertion (A) and other is Reason (R). Each of these questions also has four alternative choices, only one of which is the correct answer. You have to select one of the codes (a), (b), (c) and (d) given below.

Codes
(a) Both A and R are individually true and R is the correct explanation of A.
(b) Both A and R are individually true but R is not the correct explanation of A.
(c) A is true but R is false.
(d) A is false but R is true.

45. Assertion (A) \( \{x \in R \mid x^2 < 0\} \) is not a set. Here, \( R \) is the set of real numbers.

Reason (R) For every real number \( x \), \( x^2 \geq 0 \). \( \text{(NDA 2008 I)} \)

46. Assertion (A) If \( A = \{1, 2, 3, 4\} \), then the relation \( R \) defined on \( A \) is \( \{(1, 1), (2, 2), (3, 3), (4, 4)\} \) is transitive.

Reason (R) A relation \( R \) defined on \( A \) is transitive iff \( aRb, bRc \Rightarrow cRa \forall a, b, c \in A \).

47. Assertion (A) If \( A = \{1, 2, 3\} \), \( B = \{2, 4\} \), then the number of relation from \( A \) to \( B \) is equal to 26.

Reason (R) The total number of relation from set \( A \) to set \( B \) is equal to \( 2^{n(A) \times n(B)} \).

Directions (Q. Nos. 48-51) Consider a relation \( R \) is defined from a set \( A = \{2, 3, 4, 5\} \) to a set \( B = \{3, 6, 7, 10\} \) as follows \( (x, y) \in R \Leftrightarrow x \) divides \( y \).

48. Express \( R \), as a set of ordered pairs is

(a) \( \{(2, 4), (2, 3)\} \)  
(b) \( \{(3, 2), (3, 7), (3, 9)\} \)  
(c) \( \{(2, 6), (2, 10), (3, 3), (3, 6), (5, 10)\} \)  
(d) None of the above

49. The domain of \( R \) is

(a) \( \{2, 3, 5\} \)  
(b) \( \{1, 2\} \)  
(c) \( \{2, 3\} \)  
(d) None of these

50. The range of \( R \) is

(a) \( \{3, 6, 10\} \)  
(b) \( \{1, 2\} \)  
(c) \( \{2, 3\} \)  
(d) \( \{1, 3\} \)

51. The inverse relation \( R^{-1} \) is

(a) \( \{(6, 2), (10, 2), (3, 3), (6, 3), (10, 5)\} \)  
(b) \( \{2, 4\} \)  
(c) \( \{(3, 2), (1, 3), (4, 5)\} \)  
(d) None of the above

Directions (Q. Nos. 52-55) Consider the universal set \( S = \{0, 1, 2, 3, 4, 5, 6, 7, 8, 9\} \) and \( A = \{1, 2, 3, 4\} \), \( B = \{2, 3, 5, 6\} \), \( C = \{2, 3, 7\} \), then

52. The value of \( A \) is

(a) \( \{0, 5, 6, 7, 8, 9\} \)  
(b) \( \{2, 3\} \)  
(c) \( \{5, 6\} \)  
(d) \( \{7, 8\} \)

53. The value of \( A - B \) is

(a) \( \{0, 2, 3, 5, 6, 7, 8, 9\} \)  
(b) \( \{1, 2, 5\} \)  
(c) \( \{2, 6, 7, 8, 9\} \)  
(d) None of these

54. The value of \( A \cap B \) is

(a) \( \{2, 3\} \)  
(b) \( \{5, 6\} \)  
(c) \( \{1, 3\} \)  
(d) \( \{1, 4\} \)

55. The value of \( B - A \) is

(a) \( \{1, 4\} \)  
(b) \( \{2, 3\} \)  
(c) \( \{6, 5\} \)  
(d) \( \{4, 2\} \)

Directions (Q. Nos. 56-60) Read the following passage and give answer. \( \text{(NDA 2011 I)} \)

The students of a class are offered three languages (Hindi, English and French). 15 students learn all the three languages whereas 28 students do not learn any language. The number of students learning Hindi and English but not French is twice the number of students learning Hindi and French but not English. The number of students learning English and French but not Hindi is thrice the number of students learning Hindi and French but not English. 23 students learn only Hindi and 17 students learn only English. The total number of students learning French is 46 and the total number of students learning only French is 11.

56. How many students learn precisely two languages?

(a) 55  
(b) 40  
(c) 30  
(d) 13

57. How many students learn atleast two languages?

(a) 15  
(b) 30  
(c) 45  
(d) 55

58. What is the total strength of the class?

(a) 124  
(b) 100  
(c) 96  
(d) 66

59. How many students learn English and French?

(a) 30  
(b) 43  
(c) 45  
(d) 73

60. How many students learn atleast one language?

(a) 45  
(b) 51  
(c) 96  
(d) None of these

61. For a set \( A \), consider the following statements

I. \( A \cup P(A) = P(A) \)  
II. \( A \cap P(A) = A \)  
III. \( P(A) - A = P(A) \)

Where \( P \) denotes power set.

Which of the statements given above is/ are correct?

(a) I only  
(b) II only  
(c) III only  
(d) I, II and III
Level I

1. If \((a, b) \in A \times B\), then \(a \in A, b \in B\), so \(a = 1, 2\)
   
   \(A = \{1, 2\}\)

2. \((A \cup B) = \{1, 2, 3, 4\}\) and \(A \cap B = \{3\}\)

\[\therefore (A \cup B) \times (A \cap B) = \{(1, 3), (2, 3), (3, 3), (4, 3)\}\]

3. If \(A = B\), then \(A \cap B = A \cup B\), so \(A \cap B \subseteq A \cup B\).

4. \(n(A) = n(A) - n(A \cap B) = 8 - 2 = 6\)

5. The power set is a set of all subsets. So, it contain \(2^n\) elements.

6. Given, \(n(A) = 4, n(B) = 3\)

   We know that, the total number of relations from two finite sets \(A\) to \(B\) is given by \(2^{n(A) \times n(B)} = 2^{4 \times 3} = 2^9\).

7. If \(\phi\) is a null set, then its other representation is \(\{\}\).

   \(\therefore \phi = \{\}\)

8. Given, \(A = \{a, b, c\}\)

   Number of subsets of \(A = 2^n = 2^3 = 8\)

   \(\{\text{where, } n = \text{number of elements in } A, n = 3\}\)

   \(\therefore \text{Proper subset of } A = 2^n - 1 = 8 - 1 = 7\)

9. \(A = \{1, 2, 5, 6\}\) and \(B = \{1, 2, 3\}\)

\(\therefore A \times B = \{(1, 1), (1, 2), (1, 3), (2, 1), (2, 2), (2, 3), (5, 1), (5, 2), (5, 3), (6, 1), (6, 2), (6, 3)\}\)

   \(\text{and } B \times A = \{(1, 1), (1, 2), (1, 5), (1, 6), (2, 1), (2, 2), (2, 5), (2, 6), (3, 1), (3, 2), (3, 5), (3, 6)\}\)

   \(\Rightarrow (A \times B) \cap (B \times A) = \{(1, 1), (1, 2), (2, 1), (2, 2)\}\)

10. We have, \(A = B \cup C\)

\(\therefore E = (E - (E - (E - (E - A))))\)

\[= E - (E - (E - A))\]

\[= E - (E - A)\]

\[= E - (E - A^c) = E - A^c\]

\(\therefore A^c = B^c \cap C^c\)

Level II

11. Given that, there are two sets \(A\) and \(B\), which contains 3 and 6 elements each.

   \(\therefore\) We have, \(n(A \cup B) = n(A) + n(B) - n(A \cap B)\)

   \(= 3 + 6 - 0 = 9\) \(\therefore n(A) = 3, n(B) = 6\)

   or \(n(A \cup B) = 3 + 6 = 9\)

   or \(n(A \cup B) = 3 + 6 - 2 = 7\)

   or \(n(A \cup B) = 3 + 6 - 3 = 6\)

   \(\therefore\) These are 4 possibilities of \(n(A \cap B)\)

12. Total number of students = 100

   Number of students having Science = 70

   Number of students having Mathematics = 60

   Number of students having both Science and Mathematics = 40

   \(\therefore\) Now, number of students having Science only

   \(= 70 - 40 = 30\)

   \(\therefore\) Number of students having Mathematics only

   \(= 60 - 40 = 20\)

   Thus, number of students having Science only, Mathematics only and both subjects = 30 + 20 + 40 = 90

   Number of students, who have not taken Science Mathematics both of the subjects

   \(= 100 - 90 = 10\)

13. We have, \(A = \{1, 2, 3, 4\}\)

   and \(R = \{(1, 1), (1, 3), (2, 2), (2, 3), (3, 1), (3, 3)\}\)

   (i) \(R\) is not reflexive, since \((3, 3) \in R\)

   (ii) \(R\) is symmetric, since for each \((a, b) \in R\)

   we have \((b, a) \in R\)

   (iii) \(R\) is not transitive, since for any \((a, b) \in R\)

   and \((b, c) \in R\), we do not find \((a, c) \in R\)

   (iv) \(R\) is not anti-symmetric, since for any \((a, b) \in R\) and

   \((b, a) \in R\), we do not have \(a = b\)

   \(e.g., (1, 3) \in R, (3, 1) \in R\) but \(1 \neq 3\).

   Hence, \(R\) is only symmetric.
14. ∴ A, B and C are non-empty sets, such that A ∩ C = φ.  
   ∴ (A × B) ∩ (C × B) = (A ∩ C) × B = φ × B = φ  
15. ∴ A = {4n + 2 : n ∈ N}  
   and  
   B = {3n : n ∈ N}  
   ∴ A ∩ B = {6, 10, 14, 18, 22, 26, 30, ...}  
   and  
   = {3, 6, 9, 12, 15, 18, 21, 24, ...}  
   ∴ A ∩ B = φ  
   and  
   = φ.  
16. ∴ A ∩ B = φ (given)  
   ∴ A − B = A − (A ∩ B)  
   (∵ A − B = A)  
   Now,  
   B − A = φ  
   and  
   or  
   B − A = B ∩ A  
   and  
   (A − B) ∩ B = A ∩ B  
   17. Since, the sets A and B are not known, then cardinality of the set A ∩ B cannot be determined.  
18. If A and B are finite sets, then  
   n(A ∩ B) = n(A) − n(A ∩ B)  
19. ∴ A = {1, 2, 3} and R = {(1, 1), (2, 2), (3, 3)}  
   This relation is reflexive, since 1R1, 2R2 and 3R3.  
20. We have, X = {multiples of 2}, Y = {multiples of 5} and Z = {multiples of 10}  
   ∴ X ∩ (Y ∩ Z) = X ∩ Z  
   = {multiples of 10}  
   = {multiples of 10} = Z  
   ∴ X ∩ Y ∩ Z = Z  
21. Since, number of elements in X be n, then the number of relations on X and 2^X.  
22. |x| < 1 ⇒ −1 < x < 1 (In this interval no natural number will satisfy it). So, it is a null set.  
23. ∴ n(A) = 3 and n(B) = 2  
   ∴ Number of relations from A to B = 2^(n(A) × n(B))  
   = 2^(3 × 2) = 2^6 = 64  
24. Since, intelligence is not defined for students in a class.  
   i.e., Not a well defined collection.  
25. A = P((1, 2)) = {∅, {1}, {2}, {1, 2}}  
   From above, it is clear that {1, 2} ∈ A  
26. Here, A and B are any two sets and U be the universal set.  
27. A = {3, 5, 9, 1}, B = {1, 2, 3, 4, 5, 6, 7, 8, 9}  
   Clearly, option (d) is not correct.  
28. y = e^x and \( y = e^{-x} \) only common solution at (0, 1), so  
   A ∩ B is singleton set.  
29. A = {−2, −1, 0, 1, 2},  
   Since, R = {(x, y): y = |x|}  
   ∴ −2R1, −1R0, 0R1, 1R0 and 2R2 satisfies relation  
   y = |x| on A.  
30. A = {2, 3, 4, 5}, if a ∈ A, then (a, a) ∈ R for every a. So, R is reflexive.  
31. ∴ A ∪ B = B ∩ C  
   From above strong inference is A ⊆ B ⊆ C.  
   e.g., A = {a}, B = {a, b}, C = {a, b, c}  
   A ∪ B = {a, b}, B ∩ C = {a, b}  
32. Given function, f(x) = \( \frac{x}{x^2 + 1} \)  
   For the function f(x) is one-one  
   \( f(x_1) = f(x_2) \)  
   \[ \Rightarrow x_1 = x_2 \]  
   \[ \Rightarrow x_1^2 + 1 = x_2^2 + 1 \]  
   \[ \Rightarrow x_1^2 = x_2^2 \]  
   \[ \Rightarrow x_1 = x_2 \]  
   So, the function is one-one.  
   Let  
   \[ y = \frac{x}{x^2 + 1} \]  
   \[ \Rightarrow yx^2 + y = x \]  
   \[ \Rightarrow y = \frac{x^2 - y}{x^2 + 1} \]  
   \[ \Rightarrow y = \frac{1 \pm \sqrt{1 - 4y^2}}{2y} \]  
   \[ \Rightarrow 1 - 4y^2 \geq 0 \]  
   \[ (1 - 2y)(1 + 2y) \geq 0 \]  
   \[ (2y - 1)(2y + 1) \leq 0 \]  
   \[ y \in \left[ -\frac{1}{2}, \frac{1}{2} \right] \sim \{0\} \]  
   Here, the range of f(x) = \( \left[ -\frac{1}{2}, \frac{1}{2} \right] \sim \{0\} \)  
   and codomain = R  
   \[ \Rightarrow \text{Range} \neq \text{Codomain} \]  
   ∴ f(x) is not onto.  
   Hence, f(x) is one-one but not onto.  
33. Given, A = {−1, 2, 5, 8} and B = {0, 1, 3, 6, 7}  
   ∴ R = {(−1, 0), (2, 3), (5, 6)}  
   (∵ R = A is one less than from B)  
   Hence, total number of elements in R is 3.  
34. Now, n(A − B) = n(A) − n(A ∩ B)  
   \[ \Rightarrow 47 = 115 - n(A ∩ B) \]  
   \[ \Rightarrow n(A ∩ B) = 68 \]  
   ∴ n(A ∪ B) = n(A) + n(B) − n(A ∩ B)  
   \[ = 115 + 326 - 68 = 373 \]  
35. The number of elements in power set of A is 1.  
   \[ \Rightarrow P(P(A)) = 2^9 = 1 \]  
   ∴  
   \[ P(P(P(A))) = 2^2 = 4 \]  
   \[ = P(P(P(A))) = 2^4 = 16 \]
36. Given, \( N_a = \{ax | x \in N\} \)
   \[ \therefore N_{12} = \{12, 24, 36, 48, \ldots \} \]
   and \( N_8 = \{8, 16, 24, \ldots \} \)
   \[ \therefore N_8 \cap N_{12} = \{24, 48, \ldots \} = N_{24} \]

37. From the given figure clearly,
   Area of shaded portion = \((A \cup B)^c + (A \cap B)\)
   We are given
   \[ |E| = 42, |A| = 15, |B| = 12 \text{ and } |(A \cup B)| = 22 \]
   Now, \((A \cup B)^c = |E| - (A \cup B)| = 42 - 22 = 20 \]
   Also, we know that
   \[ |(A \cap B)| = |A| + |B| - |(A \cap B)| \]
   \[ \Rightarrow |(A \cap B)| = 15 + 12 - 22 = 5 \]
   \[ \therefore \text{Area of shaded portion} = 20 + 5 = 25 \]

38. Given that, \( n(U) = 700, n(A) = 200, n(B) = 300 \)
   and \( n(A \cap B) = 100 \)
   We know that,
   \[ n(A \cup B) = n(A) + n(B) - n(A \cap B) \]
   \[ = 200 + 300 - 100 = 400 \]
   Now,
   \[ n(A' \cap B') = n(U) - n(A \cup B) \]
   \[ = 700 - 400 = 300 \]

39. Given,
   \( A = \{1, 4, 9, 16, 25, 36, 49, 64, 81\} \)
   and \( B = \{2, 4, 6, \ldots \} \)
   Now,
   \( A \cap B = \{4, 16, 36, 64\} \)
   \[ \therefore \text{The cardinality of } (A \cap B) \]
   \[ = \text{Number of elements in } (A \cap B) = 4 \]

40. \( X = \{4^n - 3n - 1 | n \in N\} \)
    and \( Y = \{9(n - 1) | n \in N\} \)
    \[ \Rightarrow X = \{0, 9, 54, \ldots \} \]
    and \( Y = \{0, 9, 18, 27, 36, 54, \ldots \} \)
    \[ \therefore X \cup Y = \{0, 9, 18, 27, 36, 45, \ldots \} = Y \]

41. The total number of elements common in \((A \times B)\) and \((B \times A)\) is \(n^2\).

42. Total number of proper subsets of a finite set with \(n\) elements is \(2^n - 1\).

43. \( \therefore \) Given that, \( A \cup B = A \cap C \)
    and \( A \cap B = A \cap C \)
    Then, let \( A = \{a, b\}, B = \{a, c\} \)
    and \( C = \{a, c\} \)

Level II

1. We know that by the definition of subsets, if \( A = B \)
   Then, \( A \cup B = A \cap B, \forall A, B \in X \)
   \[ \therefore n(A \cup B) = n(A) + n(B) - n(A \cap B) \]
   and by De-Morgan’s law \((A \cup B)^c = A^c \cap B^c \)

2. For first statement i.e., all poets (\( P \)) are learned people (\( L \)) and from second statement all poets who learned people (\( L \)) are happy (\( H \)),
   \[ \Rightarrow A \cup B = \{a, b, c\}, A \cap C = \{a, b, c\} \]
   \[ \therefore C = \{a, b, c\} \]

3. \( R \) is relation defined on \( N \times N \) by \((a, b) R (c, d)\) if and only if \( ad = bc \), then \( R \) is
   (i) Reflexive Since, for any \( a, b \in N \)
   \[ \Rightarrow ab = ba \Rightarrow (a, b) R (a, b) \]
   (ii) Symmetric Let \((a, b) R (c, d) \Rightarrow ad = bc \Rightarrow bc = ad \)
   \[ \Rightarrow cb = da \quad (\because \text{commutativity of natural numbers}) \]
   \[ \Rightarrow (c, d) R (a, b) \]
   (iii) Transitive Let \((a, b) R (c, d) \) and \((c, d) R (e, f) \)
   \[ \Rightarrow ad = bc \text{ and } cf = de \]
   \[ \Rightarrow ade = bce \Rightarrow acf = bce \]
4. \( A = \{1, 2, 3, 4\} \) and \( B = \{2, 3, 5\} \n\)
Now, \((x, y)\) ordered pair of \( A \times B \), where \( x \in A \) and \( y \in B \).
\( \)
The ordered pairs \((x, y)\) such that \( x < y \) are
\((1, 2), (1, 3), (1, 5), (2, 3), (2, 5), (3, 5), (4, 5)\).
5. \( f(x) = \cos 2x - \sin 2x \)
\( \therefore f(x) = a \cos x + b \sin x - \sqrt{a^2 + b^2} \leq f(x) \leq \sqrt{a^2 + b^2} \)
\( -\sqrt{1 + 1} \leq \cos 2x - \sin 2x \leq \sqrt{1 + 1} \)
\(-\sqrt{2} \leq \cos 2x - \sin 2x \leq \sqrt{2} \)
So, Range of \( f(x) \) is \([-\sqrt{2}, \sqrt{2}]\).
6. Union of the closed sets \( \left[ 2 + \frac{1}{n}, 10 - \frac{1}{n} \right] \), \( n = 1, 2, \ldots \) is \((2, 10)\).
7. \( Q = \sin \theta + \cos \theta \)
\( \)
Which is of the form \( Q = a \sin \theta + b \cos \theta \), then
\(-\sqrt{a^2 + b^2} \leq Q \leq \sqrt{a^2 + b^2} \)
\( \Rightarrow -\sqrt{1 + 1} \leq Q \leq \sqrt{1 + 1} \)
\( \therefore -\sqrt{2} \leq Q \leq \sqrt{2} \)
8. The shaded portion represents in the given figure is \((P \cap Q) - R \) \( \cup \) \((P \cap R) - Q\).
9. Since, on the set of real numbers, \( R \) is a relation defined by \( xRy \) if and only if \( 3x + 4y = 5 \) for which \( 1R \frac{1}{2} \) and \( \frac{2}{3} R \frac{3}{4} \).
\( \)
i.e., \( 1R \frac{1}{2} \Rightarrow 3 \times 1 + 4 \times \frac{1}{2} = 5 \)
\( \)and \( \frac{2}{3} R \frac{3}{4} \Rightarrow 3 \times \frac{2}{3} + 4 \times \frac{3}{4} = 5 \)
\( \)
Hence, both the statements II and III are correct.
10. \( \because M = \) Set of men and \( R \) is a relation is son of defined on \( M \).
\( \)
Reflexive relation \( aRa \).
\( \)
Since, \( a \) cannot be a son of \( a \).
\( \)
Symmetric relation \( aRa \Rightarrow bRa \)
\( \)
which is also not possible.
\( \)
Transitive relation \( aRb, aRc \Rightarrow cRa \)
\( \)
which is not possible.
11. \( A = \{n^2 : n \in N\} \) and \( B = \{n^3 : n \in N\} \)
\( A = \{1, 4, 9, 16, 25, 49, 64, 81, \ldots \} \)
\( B = \{1, 8, 27, 64, 125, \ldots \} \)
\( A \cap B = \{1, 64, \ldots \} \)
\( \therefore A \cap B \) must be a proper subset of \( \{m^6 : m \in N\} \)
12. \( a) \Rightarrow R = \{(1, 1), (2, 2), (3, 3), (1, 2), (2, 3), (1, 3)\} \)
\( \)
Reflexive
\( \therefore 1R1, 2R2, 3R3 \)
\( \therefore R \) is a reflexive relation.

\[ \therefore 1R2 \text{ but } 2R1 \]
\( \cdot \)
\( \therefore R \) is not a symmetric relation.

\[ \therefore 1R2, 2R3 \Rightarrow 1R3 \]
\( \cdot \)
\( \therefore R \) is a transitive relation.
13. Given that, \( xSy \) is defined relation, if \( x^2 + y^2 = 1 \) and \( ySx \) is defined relation if \( y^2 + x^2 = 1 \)
\( \Rightarrow x^2 + y^2 = 1 \)
\( \therefore xS y \Rightarrow ySx \)
\( \cdot \)
\( \therefore R \) is a symmetric relation.
14. \( F(n) = \) Set of all divisor of a natural number \( n \).
\( \therefore F(20) = \{1, 2, 4, 5, 10\}, F(16) = \{1, 2, 4, 8\} \)
\( \therefore F(20) \cap F(16) = \{1, 2, 4\} \)
\( \therefore f(y) = \{1, 2, 4\} \)
\( \cdot \)
\( \therefore \) set of all divisor of a natural number \( y \)
\( \therefore y = 4 \)
15. In the given figure marked region represents the people who can speak Hindi, English and Kannada only.
16. It is clear from the graph that \( f(x) = e^x, \forall x \in R \)
\( \therefore R \) is many-one onto.

\[ \therefore f(x) = \text{Codomain} \]
\( \therefore f(x) \) is onto.
Hence, \( f(x) \) is many-one onto.
17. Since, on taking a straight line parallel to \( x \)-axis, the group of given function intercept it at many points.
\( \therefore f(x) \) is many-one.
And as range of \( f(x) \) intersect it at many points.
\( \therefore f(x) \) is onto.
Hence, \( f(x) \) is many-one onto.
18. An injective function means one-one.
\( \therefore f(x) = -x \)
\( \therefore \) For every values of \( x \), we get a different value of \( f \).
Hence, it is injective.
19. \( \therefore \) \( (a, a), (b, b), (c, c) \in R \)
\( \therefore R \) is a reflexive relation.
But \( (a, b) \in R \) and \( (b, a) \notin R \).
\( \therefore R \) is not a symmetric relation.
Also, \( (a, b), (b, c) \in R \)
\( \Rightarrow (a, c) \notin R \)
\( \therefore R \) is not a transitive relation.
20. Let us consider \( a \) and \( b \in (a \cap \beta) \times (\xi \cap \eta) \)
\( \Rightarrow a \in (a \cap \beta) \) and \( b \in (\xi \cap \eta) \)
\( \Rightarrow \) \((a \in \alpha) \) and \((a \in \beta) \) and \((b \in \xi) \) and \((b \in \eta) \)
\( \Rightarrow \) \((a \in \alpha) \) and \((b \in \beta) \) and \((a \in \xi) \) and \((b \in \eta) \)
21. In a Euclidean plane, orthogonality of lines is not an equivalence relation.

22. We define $AB \parallel BC$ by the open sentence $A$ is cardinally equivalent to $B$ on the family of sets. Then, the relation $E_r$ on family of set is an equivalence relation, which is true.

23. Let $y = 2x + 5$

\[ y - 5 = 2x \Rightarrow x = \frac{y - 5}{2} = g^{-1}(y) \]

\[ g^{-1}(x) = \frac{x - 5}{2} \]

24. Since, $3 \in A$

But $(3, 3) \in R$

So, it is not reflexive.

and $(3, 4) \in R$ and $(4, 3) \in R$

But $(3, 3) \in R$

So, it is also not transitive.

Hence, $R$ is neither reflexive nor transitive.

25. Since, $f(-1) = f(1) = 2^{35}$

i.e., two real numbers $1$ and $-1$ have the same image.

So, the function is not one-one and let

\[ y = (x^2 + 1)^{35} \Rightarrow x = \sqrt[35]{y - 1} \]

Thus, every real number has no pre image. So, the function is not onto.

Hence, the function is neither one-one nor onto.

26. \[
\Rightarrow R = \{(x, y) \mid xy > 0, x, y \in N\} \\
\text{Reflexive} \\
\Rightarrow x, y \in N \Rightarrow x = 0 \Rightarrow x^2 > 0 \\
\Rightarrow R \text{ is reflexive.} \\
\text{Symmetric} \\
\Rightarrow x, y \in N \text{ and } xy > 0 \Rightarrow yx > 0 \\
\Rightarrow R \text{ is also symmetric.} \\
\text{Transitive} \\
\Rightarrow x, y, z \in N \Rightarrow xy > 0, yz > 0 \Rightarrow xz > 0 \\
\Rightarrow R \text{ is also transitive.} \\
\text{Thus, } R \text{ is an equivalence relation.} \\
\]

27. For reflexive

\[ aRa \Rightarrow a \text{ divides } a \]

\[ \therefore R \text{ is reflexive.} \]

For symmetric

\[ aRb \Rightarrow a \text{ divides } b \]

\[ bRa \Rightarrow b \text{ divides } a \]

which may not be possible.

\[ \therefore R \text{ is not symmetric.} \]

For transitive

\[ aRb \Rightarrow a \text{ divides } b \Rightarrow b = ka \]

\[ bRc \Rightarrow b \text{ divides } c \Rightarrow c = lb \]

Now, $c = kb$.

\[ \Rightarrow a \text{ divides } c \Rightarrow aRc \Rightarrow aRb, bRc \Rightarrow cRa \]

Thus, $R$ is transitive.

28. Both the statements are incorrect.

29. $A - P(A) = A$

Which is correct as $A$ and $P(A)$ are disjoint sets.

30. We know that,

\[
[(A \cup B) \cap C'] = (A \cup B)' \cup C' = (A' \cap B') \cup C' = A' \cap B' \cup C' \text{ (by De-Morgan’s law)}
\]

31. Let us consider an example.

Let $A = \{a, b, c\}$ and $B = \{a, b, c, d\}$

Here, 3 elements are common in $A$ and $B$.

Now,

\[ (A \cap B) = \{a, b, c\}, (B \cap A) = \{a, b, c\}, (A \cap B \cap C) = \{a, b, c\} \]

\[ \text{Here common element in } (A \times B) \text{ and } (B \times A) \text{ is } 9 \text{ i.e., } (3)^2 \]

So, in general, if $A$ and $B$ are two non-empty sets having '$n'$ elements in common, then $(n)^2$ is the common elements in the sets $A \times B$ and $B \times A$.

32. $A = \{-1, 1\}, B = \{-1, 1, -i, i\}$

\[ A - B = \emptyset, B - A = \{-i, i\} \]

\[ \therefore (A - B) \cup (B - A) = \{-i, i\} \]

33. \[ n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(AB) - n(BC) - n(CA) + n(ABC) \]

\[ = 80 \text{ (number of families reading atleast one newspapers } A, B \text{ and } C) \]

\[ \therefore \text{ Total number of families } = 100 \]

So, 20 families do not read any newspaper.

34. $2^{mn} = 64 \Rightarrow mn = 6$

\[ \therefore \text{ The possible value of } m \text{ and } n \text{ are } 2 \text{ and } 3. \]

35. If elements are not repeated, then number of elements in $A_1 \cup A_2 \cup \ldots \cup A_9$ is $30 \times 5$. But each element is used 10 times so, $\frac{30 \times 5}{10} = 15$. Similarly, if elements in $B, B_1, B_2, \ldots, B_n$ are not repeated, then total number of elements is $3n$ but each element is repeated 9 times so, $\frac{3n}{9} = 15 \Rightarrow n = 45$.

36. $U = \{x : x^2 - 6x + 11 + 11x^2 - 6x^2 = 0\} = \{0, 1, 2, 3\}$

$A = \{x : x^2 - 5x + 6 = 0\} = \{2, 3\}$

and $B = \{x : x^2 - 3x + 2 = 0\} = \{2, 1\}$

$A \cap B = \{2\}$

\[ \therefore (A \cap B)' = U - (A \cap B) = \{0, 1, 2, 3\} - \{2\} = \{0, 1, 3\} \]

37. $R$ is a relation defined on the set of 2 integers as follows

\[ mRn \iff m + n \text{ is odd.} \]

(I) We know that the sum of two odd and even numbers is an even number. Thus, it may be reflexive or not.

(II) If $m$ and $n$ are numbers, such that

\[ mRn \iff m + n \text{ is odd.} \]
Sets, Relations and Functions

Thus, \( nRm \iff n + m \) is odd.

Therefore, only (II) statement is correct.

38. \( R \) is non-anti-symmetric.

39. \( A = \{(x, y) \mid x + y \leq 4\} \) and \( B = \{(x, y) \mid x + y \leq 0\} \)

40. The correct relation is \( (R_1 \cap R_2)^{-1} = R_1^{-1} \cup R_2^{-1} \).

41. Since, the number of elements in set \( A \) is 4.

42. \( f(x) = x \mid x \)

43. \( A \subseteq X \) and \( B \subseteq X \)

44. Given that, \( U = \{x \in N : 1 \leq x \leq 10\} \)

45. Both A and R are true and R is the correct explanation of A.

46. Since, \( (1, 1), (2, 2), (3, 3), (4, 4) \) is identity relation and identity relation is always equivalence relation. So, it is transitive.

47. We know by the property of relation, the total number of relation from set A to set B is \( 2^{\#(A) \times \#(B)} \).

48. Recall that \( a/b \) stands for ‘a divides b’. For the elements of the given sets A and B, we find that \( 2/6, 2/10, 3/3, 3/6 \) and \( 5/10 \).

49. Clearly, domain \( (R) = \{2, 3, 5\} \)

50. Range \( (R) = \{3, 6, 10\} \)

51. By definition, \( R^{-1} = \{(6, 2), (10, 2), (3, 3), (6, 3), (10, 5)\} \)

52. \( A = \{x : x \in S \text{ and } x \notin A\} = \{0, 5, 6, 7, 8, 9\} \)

53. \( A \setminus B = \{x : x \in A \text{ and } x \notin B\} = \{1, 4\} \)

54. \( A \setminus B = \{0, 5, 6, 7, 8, 9\} \cap \{2, 3, 5, 6\} = \{5, 6\} \)

55. \( B = S - B = S \setminus \{2, 3, 5, 6\} = \{0, 1, 4, 7, 8, 9\} \)

56. The number of students learning precisely two languages

57. The number of students learning at least two languages

58. The total strength of the class

59. The number of students learn English and French

60. The number of students learn at least one of the languages

61. Let \( A = \{1, 2\} \) and \( \{A\} = \{\{1, 2\}\} \)

62. \( P(A) = \{\{1\}, \{2\}, \{1, 2\}, \emptyset\} \)

63. Then, \( A \cup P(A) \neq P(A) \)

64. \( \{A\} \cap P(A) = \{\{1, 2\}\} \cap \{\{1\}, \{2\}, \{1, 2\}, \emptyset\} \)

65. \( \{1, 2\} = A \)