# CAT SOLVED PAPER

## Fresh Paper

*No. of Questions :* 50 *Time :* 40 *min*

Note Each wrong answer carry $\frac{1}{3}$rd negative mark.

✍ ***Directions for question number* 1 *to* 5 *:*** *Answer the questions independently of each other.*

**1.** The infinite sum $1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}+\ldots$ equals :

(a) $\frac{27}{14}$ (b) $\frac{21}{13}$

(c) $\frac{49}{27}$ (d) $\frac{256}{147}$

**2.** Consider the sets $T_n=\{n, n+1, n+2, n+3, n+4\}$, where $n=1, 2, 3, \ldots 96$. How many of these sets contain 6 or any integral multiple there of (*i.e.*, any one of the numbers 6, 12, 18 ...... ) ?

(a) 80 (b) 81

(c) 82 (d) 83

**3.** Let *ABCDEF* be a regular hexagon. What is the ratio of the triangle *ACE* to that of the hexagon *ABCDEF* ?

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$

(c) $\frac{2}{3}$ (d) $\frac{5}{6}$

**4.** The number of roots common between the two equations $x^3+3x^2+4x+5=0$ and $x^3+2x^2+7x+3=0$ :

(a) 0 (b) 1

(c) 2 (d) 3

**5.** A real number $x$ satisfying $1-\frac{1}{n}<x\le 3+\frac{1}{n}$, for every positive integer $n$, is best described by :

(a) $1<x<4$ (b) $1<x\le 3$

(c) $0<x\le 4$ (d) $1\le x\le 3$

✍ ***Directions for question number* 6, 7, *and* 8 *:*** *Answer the questions on the basis of the tables given below.*

*Two binary operations* $\oplus$ *and* * *are defined over the set* $\{a, e, f, g, h\}$ *as per the following tables :*

| $\oplus$ | a | e | f | g | h |
|---|---|---|---|---|---|
| a | a | e | f | g | h |
| e | e | f | g | h | a |
| f | f | g | h | a | e |
| g | g | h | a | e | f |
| h | h | a | e | f | g |

| * | a | e | f | g | h |
|---|---|---|---|---|---|
| a | a | a | a | a | a |
| e | a | e | f | g | h |
| f | a | f | h | e | g |
| g | a | g | e | h | f |
| h | a | h | g | f | e |

*Thus according to the first table* $f\oplus g=a$, *while according to the second table* $g * h=f$, *and so on.*

*Also, let* $f^2=f*f$, $g^3=g*g*g$, *and so on.*

**6.** What is the smallest positive integer $n$ such that $g^n=e$ ?

(a) 4 (b) 5

(c) 2 (d) 3

**7.** Upon simplification $f\oplus[f*\{f\oplus(f*f)\}]$ equals :

(a) $e$ (b) $f$

(c) $g$ (d) $h$

**8.** Upon simplification $\{a^{10}*(f^{10}\oplus g^9)\}\oplus e^8$ equals :

(a) $e$ (b) $f$

(c) $g$ (d) $h$

✍ ***Directions for question number of* 9 *and* 10 *:*** *Answer the questions on the basis of the information given below.*

*A string of three English letters is formed as per the following rules :*

*(a) The first letter is any vowel.*

*(b) The second letter is m, n or p.*

*(c) If the second letter is m then the third letter is any vowel which is different from the first letter.*

*(d) If the second letter is n then the third letter is e or u.*

*(e) If the second letter is p then the third letter is the same as the first letter.*

**9.** How many strings of letters can possibly be formed using the above rules ?

(a) 40 (b) 45

(c) 30 (d) 35

**10.** How many strings of letters can possibly be formed using the above rules such that the third letter of the string is $e$ ?

(a) 8 (b) 9

(c) 10 (d) 11

✍ ***Directions for question number* 11 *to* 15 *:*** *Answer the questions independently of each other.*

**11.** What is the remainder when $4^{96}$ is divided by 6 ?

(a) 0 (b) 2

(c) 3 (d) 4

**12.** If $x$ and $y$ are integers then the equlion $5x+19y=64$ has :

(a) no solution for $x<300$ and $y<0$

(b) no solution for $x>250$ and $y>-100$

(c) a solution for $250<x<300$

(d) a solution for $-59<y<-56$

**13.** If $\frac{1}{3}\log_3 M+3\log_3 N=1+\log_{0.008} 5$, then :

(a) $M^9=\frac{9}{N}$ (b) $N^9=\frac{9}{M}$

(c) $M^3=\frac{3}{N}$ (d) $N^9=\frac{3}{M}$

**14.** Using only 2, 5, 10, 25 and 50 paise coins, what will be the minimum number of coins required to pay exactly 78 paise, 69 paise and Re. 1.01 to three different persons ?

(a) 19 (b) 20
(c) 17 (d) 18

**15.** The length of the circumference of a circle equals the perimeter of a triangle of equal sides, and also the perimeter of a square. The areas covered by the circle, triangle, and square are $c, t$ and $s$ respectively. Then :

(a) $s > t > c$ (b) $c > t > s$
(c) $c > s > t$ (d) $s > c > t$

✍ ***Direction for question number 16 to 18 :*** *Answer the questions on the basis of the information given below.*
*The seven basic symbols in a certain numeral system and their respective values are as follows :*
$T = 1, V = 5, X = 10, L = 50, C = 100, D = 500$ *and* $M = 1000$
*In general, the symbols in the numeral system are read from left to right, starting with the symbol representing the largest value; the same symbol cannot occur contiguously more than three times; the value of the numeral is the sum of the values of the symbols.*
*For example,* $XXVII = 10 + 10 + 5 + 1 + 1 = 27$.
*An exception to the left-to-right reading occurs when a symbol is followed immediately by a symbol of greater value; then, the smaller value is subtracted from the larger. For example,* $XLVI = (50 - 10) + 5 + 1 = 46$.

**16.** The value of the numeral *MDCCLXXXVII* is :

(a) 1687 (b) 1787
(c) 1887 (d) 1987

**17.** The value of the numeral *MCMXCIX* is :

(a) 1999 (b) 1899
(c) 1989 (d) 1889

**18.** Which of the following can represent the numeral for 1995 ?

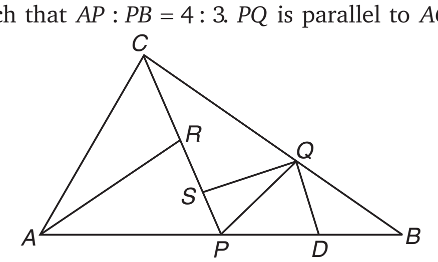
(1) *MCMLXXV* (2) *MCMXCV*
(3) *MVD* (4) *MVM*
(a) only (1) and (2) (b) only (3) and (4)
(c) only (2) and (4) (d) only (4)

✍ ***Directions for question number 19 to 32 :*** *Answer the questions independently of each other.*

**19.** There are 12 towns grouped into four zones with three towns per zone. It is intended to connect the towns with telephone lines such that every two towns are connected with three direct lines if they belong to the same zone, and with only one direct line otherwise. How many direct telephone lines are required ?

(a) 72 (b) 90
(c) 96 (d) 144

**20.** In the figure (not drawn to scale) given below, $P$ is a point on $AB$ such that $AP : PB = 4 : 3$. $PQ$ is parallel to $AC$ and $QD$ is parallel to $CP$. In $\Delta ARC$, $\angle ARC = 90°$, and in $\Delta PQS$, $\angle PSQ = 90°$. The length of $QS$ is 6 cms. What is ratio $AP : PD$ ?

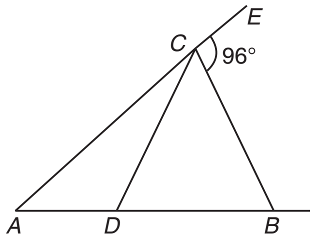



(a) 10 : 3 (b) 2 : 1
(c) 7 : 3 (d) 8 : 3

**21.** A car is being driven, in a straight line and at a uniform speed, towards the base of a vertical tower. The top of the tower is observed from the car and, in the process, it takes 10 minutes for the angle of elevation to change from 45° to 60°. After how much time will this car reach the base of the tower?

(a) $5(\sqrt{3} + 1)$ (b) $6(\sqrt{3} + \sqrt{2})$
(c) $7(\sqrt{3} - 1)$ (d) $8(\sqrt{3} - 2)$

**22.** In the figure (not drawn to scale) given below, if $AD = CD = BC$ and $\angle BCE = 96°$, how much is $\angle DBC$ ?




(a) 32°
(b) 84°
(c) 64°
(d) can't be determined

**23.** If both $a$ and $b$ belong to the set {1, 2, 3, 4}, then the number of equations of the form $ax^2 + bx + 1 = 0$ having real roots is :

(a) 10 (b) 7
(c) 6 (d) 12

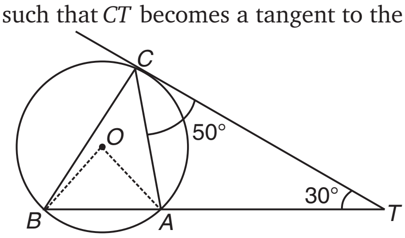
**24.** If $\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$, then a possible value of $x$ is given by :

(a) 10 (b) $\frac{1}{100}$
(c) $\frac{1}{1000}$ (d) none of these

**25.** What is the sum of all two digit numbers that give a remainder of 3 when they are divided by 7 ?

(a) 666 (b) 676
(c) 683 (d) 777

**26.** In the figure given below (not drawn to scale) $A$, $B$ and $C$ are three points on a circle with centre $O$. The chord $BA$ is extended to a point $T$ such that $CT$ becomes a tangent to the circle at point $C$. If $\angle ATC = 30°$ and $\angle ACT = 50°$, then the angle $\angle BOA$ is :




(a) 100°
(b) 150°
(c) 80°
(d) not possible to determine

**27.** What is the sum of '$n$' terms in the series :

$$\log m + \log\left(\frac{m^2}{n}\right) + \log\left(\frac{m^3}{n^2}\right) + \log\left(\frac{m^4}{n^3}\right) + \ldots$$

(a) $\log\left[\frac{n^{(n-1)}}{m^{(n+1)}}\right]^{n/2}$ (b) $\log\left[\frac{m^m}{n^n}\right]^{n/2}$
(c) $\log\left[\frac{m^{(1-n)}}{n^{(1-n)}}\right]^{n/2}$ (d) $\log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{n/2}$

**28.** Let $S_1$ be a square of side $a$. Another square $S_2$ is formed by joining the mid points of the sides of $S_1$. The same process is applied to $S_2$ to form yet another square $S_3$ and so on. If $A_1, A_2, A_3 \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3 \ldots$ respectively, then the ratio $\frac{P_1 + P_2 + P_3 + \ldots}{A_1 + A_2 + A_3 + \ldots}$ equals :

(a) $\frac{2(1+\sqrt{2})}{a}$ (b) $\frac{2(2-\sqrt{2})}{a}$

(c) $\frac{2(2+\sqrt{2})}{a}$ (d) $\frac{2(1+2\sqrt{2})}{a}$

**29.** If three positive real numbers $x, y, z$ satisfy $y - x = z - y$ and $xyz = 4$, then what is the minimum possible value of $y$ ?

(a) $2^{1/3}$ (b) $2^{2/3}$

(c) $2^{1/4}$ (d) $2^{3/4}$

**30.** An intelligence agency forms a code of two distinct digits selected from 0, 1, 2, … 9 such that the first digit of the code is non zero. The code handwritten on a slip, can however potentially create confusion when read upside down. For example, the code 91 may appear as 16. How many codes are there for which no such confusion can arise ?

(a) 80 (b) 78

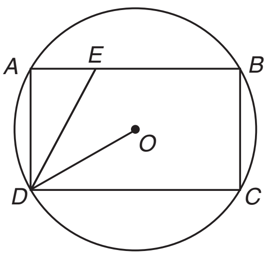
(c) 71 (d) 69

**31.** Consider two different cloth cutting processes. In the first one, $n$ circular cloth pieces are cut from a square cloth piece of side $a$ in the following steps : the original square of side $a$ is divided into $n$ smaller squares, not necessarily of the same size, then a circle of maximum possible area is cut from each of the smaller squares. In the second process, only one circle of maximum possible area is cut from the square of side $a$ and the process ends here the cloth pieces remaining after cutting the circles are scrapped in both the processes. The ratio of the total area of scrap cloth generated in the former to that in the latter is :

(a) $1:1$ (b) $\sqrt{2}:1$

(c) $\frac{n(4-\pi)}{4n-\pi}$ (d) $\frac{4n-\pi}{n(4-\pi)}$

**32.** In the figure below (not drawn to scale), rectangle $ABCD$ is inscribed in the circle with centre at $O$. The length of side $AB$ is greater than that of side $BC$. The ratio of the area of the circle to the area of the rectangle $ABCD$ is $\pi : \sqrt{3}$. The line segment $DE$ intersects $AB$ at $E$ such that $\angle ODC = \angle ADE$. What is the ratio $AE : AD$ ?

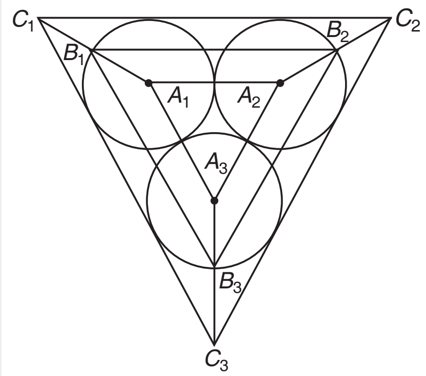



(a) $1:\sqrt{3}$ (b) $1:\sqrt{2}$

(c) $1:2\sqrt{3}$ (d) $1:2$

*Directions for question number 33 to 35 : Answer the questions on the basis of the information given below.*

*Consider three circular parks of equal size with centres at $A_1$, $A_2$ and $A_3$ respectively. The parks touch each other at the edge as shown in the figure (not drawn to scale). There are three paths formed by the triangles $A_1A_2A_3$, $B_1B_2B_3$ and $C_1C_2C_3$ as shown. Three sprinters $A$, $B$ and $C$ begin running from points $A_1$, $B_1$ and $C_1$ respectively. Each sprinter traverses her respective triangular path clockwise and returns to her starting point.*




**33.** Let the radius of each circular park be $r$, and the distances to be traversed by the sprinters $A$, $B$ and $C$ be $a$, $b$ and $c$, respectively. Which of the following is true ?

(a) $b - a = c - b = 3\sqrt{3}\,r$

(b) $b - a = c - b = \sqrt{3}\,r$

(c) $b = \frac{a+c}{2} = 2(1+\sqrt{3})r$

(d) $c = 2b - a = (2+\sqrt{3})r$

**34.** Sprinter $A$ traverse distances $A_1A_2$, $A_2A_3$, $A_3A_1$ at average speeds of 20, 30 and 15, respectively. $B$ traverses her entire path at uniform speed of $(10\sqrt{3}+20)$. $C$ traverses distance $C_1C_2$, $C_2C_3$ and $C_3C_1$ at average speeds of $\frac{40}{3}(\sqrt{3}+1)$, $\frac{40}{3}(\sqrt{3}+1)$ and 120, respectively. All speeds are in the same unit. Where would $B$ and $C$ be respectively when $A$ finishes her sprint ?

(a) $B_1$, $C_1$

(b) $B_3$, $C_3$

(c) $B_1C_3$

(d) $B_1$, somewhere between $C_3$ and $C_1$

**35.** Sprinters $A$, $B$ and $C$ traverse their respective paths at uniform speeds of $u$, $v$ and $w$ respectively. It is known that $u^2 : v^2 : w^2$ is equal to area $A$ : Area $B$ : Area $C$, where Area $A$, Area $B$ and Area $C$ are the areas of triangles $A_1A_2A_3$, $B_1B_2B_3$ and $C_1C_2C_3$, respectively. Where would $A$ and $C$ be when $B$ reaches point $B_3$ ?

(a) $A_2$, $C_3$

(b) $A_3$, $C_3$

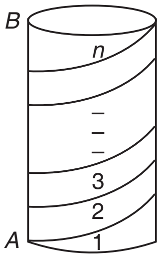
(c) $A_3$, $C_2$

(d) Somewhere between $A_2$ and $A_3$, somewhere between $C_3$ and $C_1$

*Directions for question number 36 to 38 : Answer the questions on the basis of the information given below.*

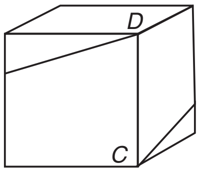
*Consider a cyclinder of height h cms and radius $r = \frac{2}{\pi}$ cms as shown in the figure (not drawn to scale). A string of a certain length when wound on its cylindrical surface starting at point A and ending at point B, gives a maximum of n turns (in other words, the string's length is the minimum length required to wind n turns).*

**36.** What is the vertical spacing in cms between two consecutive turns ?




(a) $\frac{h}{n}$
(b) $\frac{h}{\sqrt{n}}$
(c) $\frac{h}{n^2}$
(d) cannot be determined with the given information

**37.** The same string, when would on the exterior four walls of a cube of side $n$ cms, starting at point $C$ and ending at point $D$, can give exactly on turn (see figure, not drawn to scale). The length of the string, in cms, is :




(a) $\sqrt{2}n$ (b) $\sqrt{17}n$
(c) $n$ (d) $\sqrt{13}\,n$

**38.** In the setup of the previous two questions, how is $h$ related to $n$?
(a) $h = \sqrt{2}\,n$ (b) $h = \sqrt{17}\,n$
(c) $h = n$ (d) $h = \sqrt{13}\,n$

✍ ***Directions for question number 39 to 50 :*** *Answer the questions independently of each other.*

**39.** Let $x$ and $y$ be positive integers such that $x$ is prime and $y$ is composite. Then,
(a) $y - x$ cannot be an even number
(b) $xy$ cannot be an even integer
(c) $\frac{(x+y)}{x}$ cannot be an even integer
(d) none of the abvoe statements is true

**40.** A survey on a sample of 25 new cars being sold at a local auto dealer was conducted to see which of the three popular options : air conditioning, radio and power windows were already installed. The survey found :
15 had air conditioning
2 had air conditioning and power windows but no radios
12 had radio
6 had air conditioning and radio but no power windows
11 had power windows
4 had radio and power windows
3 had all three options
What is the number of cars that had none of the options ?
(a) 4 (b) 3
(c) 1 (d) 2

**41.** If $n$ is such that $36 \le n \le 72$, then

$$x = \frac{n^2 + 2\sqrt{n}(n+4) + 16}{n + 4\sqrt{n} + 4} \text{ satisfies :}$$

(a) $20 < x < 54$ (b) $23 < x < 58$
(c) $25 < x < 64$ (d) $28 < x < 60$

**42.** If $13x + 1 < 2z$ and $z + 3 = 5y^2$, then :
(a) $x$ is necessarily less than $y$
(b) $x$ is necessarily greater than $y$
(c) $x$ is necessarily equal to $y$
(d) none of the above is necessarily true

**43.** Let $n$ $(> 1)$ be a composite integer such that $\sqrt{n}$ is not an integer. Consider the following statements :
(A) $n$ has a perfect integer valued divisor which is greater than 1 and less than $\sqrt{n}$
(B) $n$ has a perfect integer valued divisor which is greater than $\sqrt{n}$ but less than $n$. Then :
(a) both A and B are false
(b) A is true but B is false
(c) A is false but B is true
(d) both A and B are true

**44.** If $|b| \ge 1$ and $x = -|a|\,b$, then which one of the following is necessarily true ?
(a) $a - xb < 0$ (b) $a - xb \ge 0$
(c) $a - xb > 0$ (d) $a - xb \le 0$

**45.** A piece of paper is in the shape of a right angled triangle and is cut along a line that is parallel to the hypotenuse, leaving a smaller triangle. There was a 35% reduction in the length of the hypotenuse of the triangle. If the area of the original triangle was 34 square inches before the cut, what is the area (in square inches) of the smaller triangle ?
(a) 16.665 (b) 16.565
(c) 15.465 (d) 14.365

**46.** In a coastal village, every year floods destroy exactly half of the huts. After the flood water recedes, twice the number of huts destroyed are rebuilt. The floods occurred consecutively in the last three years namely 2001, 2002 and 2003. If floods are given expected in 2004, the number of huts expected to be destroyed is :
(a) less than the number of huts existing at the begining of 2001
(b) less then the total number of huts destroyed by floods in 2001 and 2003
(c) less than the total number of huts destroyed by floods in 2002 and 2003
(d) more than the total number of huts built in 2001 and 2002

**47.** Let $a, b, c, d$ and $e$ be integers such that $a = 6b = 12c$ and $2b = 9d = 12e$. Then which of the following pairs contains a number that is not an integer ?
(a) $\left(\frac{a}{27}, \frac{b}{e}\right)$ (b) $\left(\frac{a}{36}, \frac{c}{e}\right)$
(c) $\left(\frac{a}{12}, \frac{bd}{18}\right)$ (d) $\left(\frac{a}{6}, \frac{c}{d}\right)$

**48.** If $a$, $a + 2$ and $a + 4$ are prime numbers, then the number of possible solutions for $a$ is :
(a) one (b) two
(c) three (d) more than three

**49.** A square tin sheet of side 12 inches is converted into a box with open top in the following steps : The sheet is placed horizontally; then equal sized squares, each of side $x$ inches, are cut from the four corners of the sheet; Finally, the four resulting sides are bent vertically upwards in the shape of a box. If $x$ is an integer, then what value of $x$ maximizes the volume of the box ?
(a) 3 (b) 4
(c) 1 (d) 2

**50.** Two straight roads $R_1$ and $R_2$ diverge from a point $A$ at an angle of 120°. Ram starts walking from point $A$ along $R_1$ at a uniform speed of 3 km/hr. Shyam starts walking at the same time from $A$ along $R_2$ at a uniform speed of 2 km/h. They continue walking for 4 hours along their respective roads and reach points $B$ and $C$ on $R_1$ and $R_2$, respectively. There is a straight line path connecting $B$ and $C$. Then Ram returns to point $A$ after walking along the line segments $BC$ and $CA$. Shyam also returns to $A$ after walking along line segments $CB$ and $BA$. Their speeds remain unchanged. The time interval (in hours) between Ram's and Shyam's return to the point $A$ is :

(a) $\frac{10\sqrt{19}+26}{3}$ (b) $\frac{2\sqrt{19}+10}{3}$

(c) $\frac{\sqrt{19}+26}{3}$ (d) $\frac{\sqrt{19}+10}{3}$

# Answers

| | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| **1** (c) | **2.** (a) | **3.** (b) | **4.** (a) | **5.** (c) | **6.** (a) | **7.** (d) | **8.** (a) | **9.** (d) | **10.** (c) |
| **11.** (d) | **12.** (c) | **13.** (b) | **14.** (a) | **15.** (c) | **16.** (b) | **17.** (a) | **18.** (c) | **19.** (b) | **20.** (c) |
| **21.** (a) | **22.** (c) | **23.** (b) | **24.** (b) | **25.** (b) | **26.** (a) | **27.** (d) | **28.** (c) | **29.** (b) | **30.** (c) |
| **31.** (a) | **32.** (a) | **33.** (a) | **34.** (c) | **35.** (b) | **36.** (a) | **37.** (b) | **38.** (c) | **39.** (d) | **40.** (d) |
| **41.** (c) | **42.** (d) | **43.** (d) | **44.** (b) | **45.** (d) | **46.** (c) | **47.** (d) | **48.** (a) | **49.** (d) | **50.** (b) |

# Hints & Solutions

**1.** Let $S = 1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \dots\infty$ ...(1)

$\therefore$ $7S = 7 + 4 + \frac{9}{7} + \frac{16}{7^2} + \frac{25}{7^3} + \dots\infty$

$\Rightarrow$ $7S = 11 + \frac{9}{7} + \frac{16}{7^2} + \frac{25}{7^3} + \dots\infty$ ...(2)

$\therefore$ $6S = 7S - S = 10 + \frac{5}{7} + \frac{7}{7^2} + \frac{9}{7^2} + \frac{11}{7^3} + \dots\infty$ [(2) – (1)]

$\therefore$ $42S = 70 + 5 + \frac{7}{7} + \frac{9}{7^2} + \frac{11}{7^3} + \dots\infty$

$\therefore$ $36S = 42S - 6S = 65 + \left[\frac{2}{7} + \frac{2}{7^2} + \frac{2}{7^3}\dots\infty\right]$

$\Rightarrow$ $36S = 65 + 2\left[\frac{\frac{1}{7}}{\frac{6}{7}}\right]$ {using sum of infinite GP}

$\Rightarrow\therefore$ $S = \frac{196}{36\times 3} = \frac{49}{27}$

$\therefore$ $S = \frac{49}{27}$

**Alternatively :** Let $S = 1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3} + \frac{25}{7^4} + \dots$

Putting $\frac{1}{7} = x$, we get

$$S = 1 + 2^2x + 3^2x^2 + 4^2x^3 + 5^2x^4 + \dots$$

$\therefore$ $S\cdot x = x + 4x^2 + 9x^2 + 16x^4 + 25x^5 + \dots$

$\therefore$ $S - Sx = 1 + 3x + 5x^2 + 7x^3 + 9x^4 + \dots$

$\therefore$ $x(S - Sx) = x + 3x^2 + 5x^3 + 7x^4 + \dots$

$\therefore$ $(S - Sx) - \{x(S - Sx)\} = 1 + 2x + 2x^2 + 2x^3 + \dots\infty$

$\Rightarrow$ $(1 - x^2)S = 1 + 2x + 2x^2 + 2x^3 + \dots\infty$

$\Rightarrow$ $(1 - x^2)S = 1 + \frac{2x}{1-x}$; $|x| < 1$

$\Rightarrow$ $S = \frac{1+x}{(1-x)^3}$

(This can be used as a direct formula for solving this type of problem.)

Now substituting $x = \frac{1}{7}$, we get

$$S = \frac{49}{27}$$

**2.** Substituting the value of $n = 1, 2, 3, \dots 96$ in $T_n$, we get the elements of $T_1, T_2, T_3 \dots T_{96}$.

*e.g.*, $T_1 = \{1, 2, 3, 4, 5\}$

$T_2 = \{2, 3, 4, 5, 6\}$

$T_3 = \{3, 4, 5, 6, 7\}$ etc.

By observation we see that for $n = 1, 7, 13, 19, 25, 31, 37, \dots 91$, there is no element in $T_1, T_7, T_{13} \dots T_{91}$ which contain 6 or any integeral multiple of 6.

Thus there are total 16 sets $(T_1, T_7, T_{13} \dots T_{91})$ out of 96 sets which do not contain any element 6 or its integral multiple.

Hence, there are only 96 – 16 = 80 sets which contain 6 or any integral multiple.

**3.** $ABCDEF$ is a regular hexagon. Joining the centre $O$ with vertices $A, E, C$, we get $\Delta AOE$, $\Delta AOC$, $\Delta EOC$.

Since $AB = BC = CD = DE = EF = FA = AO = EO = CO$

and $\angle ABC = \angle BCD = \angle CDE = \angle DEF = \angle EFA = \angle AOC$

$= \angle COE = \angle AOE = 120°$

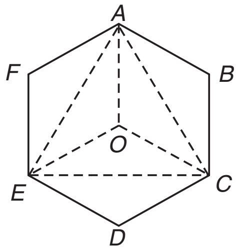
Thus the $\Delta ABC \cong \Delta AOC$

$\Delta CDE \cong \Delta COE$

$\Delta EFA \cong \Delta EOA$

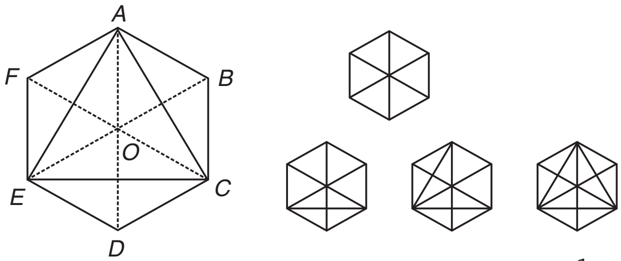
Hence, the area of $\Delta AEC$

$= \frac{1}{2}$ area of hexagon $ABCDEF$



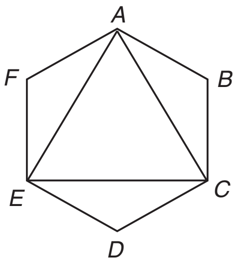

**Alternatively :** In the given figure of hexagon $ABCDEF$, there are 12 smaller congruent triangles with the angles 30° – 60° – 90°.

Out of these 12 triangles 6 triangles are contained in the larger triangle $ACE$.




Hence, the ratio of area of $\Delta ACE$ to area of hexagon $= \frac{1}{2}$

**Alternatively :**




$\because \quad \angle EDC = 120°$

$\therefore \quad \angle CED = \angle ECD = 30°$

Let each side of the hexagon be '$a$', then $EC = a\sqrt{3}$ (using cosine rule or sine rule)

$\therefore$ Area of $\Delta AEC = \frac{\sqrt{3}}{4}(a\sqrt{3})^2 = \frac{3\sqrt{3}}{4}a^2$

and area of hexagon $= 6 \times \frac{\sqrt{3}}{4} \times a^2 = \frac{6\sqrt{3}}{4}a^2$

$$\therefore \quad \frac{\text{Area of } \Delta ACE}{\text{Area of hexagon } ABCDEF} = \frac{1}{2}$$

**4.** Let $\alpha$ be the common root.

$$\therefore \quad \alpha^3 + 3\alpha^2 + 4\alpha + 5 = 0 \quad \ldots(1)$$

$$\text{and} \quad \alpha^3 + 2\alpha^2 + 7\alpha + 3 = 0 \quad \ldots(2)$$

comparing the two equations, we get

$$\alpha^3 + 3\alpha^2 + 4\alpha + 5 = \alpha^3 + 2\alpha^2 + 7\alpha + 3$$

$$\Rightarrow \quad \alpha^2 - 3\alpha + 2 = 0$$

$$\Rightarrow \quad (\alpha - 2)(\alpha - 1) = 0$$

$$\Rightarrow \quad \alpha = 1, 2$$

$\therefore$ No. of possible common roots between two equations = 2 but when $\alpha = 1, 2$ are substituted in equations (1) and (2) none of them satisfies the equations.

Hence, the number of common roots = 0

**5.** Since $n$ is a positive integer, therefore substituting the value of **least positive integer,** we get

$$1 - \frac{1}{1} < x \le 3 + \frac{1}{1} \Rightarrow 0 < x \le 4 \quad \ldots(A)$$

If ***n* is very large** *i.e.*, $n \to \infty$ then $\frac{1}{n} \to 0$

$$\therefore \quad 1 - \frac{1}{n} < x \le 3 + \frac{1}{n} \Rightarrow 1 < x \le 3 \quad \ldots(B)$$

Hence the best possible range of $x$ is described by $0 < x \le 4$.

**6.** $g^2 = g * g = h$

$g^3 = g^2 * g = h * g = f$

$g^4 = g^3 * g = f * g = e$

$\therefore \quad n = 4$

**7.** $f \oplus [f * \{f \oplus (f * f)\}] = f \oplus [f * \{f \oplus h\}] \quad (\because f * f = h)$

$= f \oplus [f * e] \quad (\because f \oplus h = e)$

$= f \oplus f \quad (\because f * e = f)$

$= h \quad (\because f \oplus f = h)$

**8.** $a^{10} = a \quad (\because a^2 = a * a = a \text{ or } a^n = a)$

Similarly, $e^8 = e$

Now, $f^2 = f * f = h$

$f^3 = f^2 * f = h * f = g$

$f^4 = f^3 * f = g * f = e$

$f^5 = f^4 * f = e * f = f$

$f^{10} = f^5 * f^5 = f * f = h$

**Alternatively :** $f^{10} = f^2 * f^2 * f^2 * f^2 * f^2$

$= h * h * h * h * h$

$= h^2 * h^2 * h$

$= e * e * h$

$= e * h = h$

Similarly, $g^9 = h$

$\therefore \quad \{a^{10} * (f^{10} \oplus g^9)\} \oplus e^8 = \{a * (h \oplus h)\} \oplus e$

$= \{a * g\} \oplus e = a \oplus e = e$

**9.** There are 3 exclusive cases :

(i) when $m$ is the second letter

(ii) when $n$ is the second letter

(iii) when $p$ is the second letter

**Case (i) :** First letter can be selected in 5 ways out of 5 vowels. Since second letter is fixed (*i. e.*, $m$) therefore no. of ways of selection of second letter is 1.

Third letter can be selected in 4 ways out of remaining 4 vowels (since the vowel which has been used at first place can not be used at the place of third letter.

$\therefore$ Total no. of ways $= 5 \times 1 \times 4 = 20$

**Case (ii) :** First letter can be selected in 5 ways.

Second letter can be selected in 1 way.

Third letter can be selected in 2 ways out of $e$, $u$.

$\therefore$ Total no. of ways $= 5 \times 1 \times 2 = 10$

**Case (iii) :** First letter can be selected in 5 ways.

Second letter can be selected in 1 way.

Third letter can be selected in 1 way since third letter will be same as the first letter. So if the first letter is selected, then there is no need to select the third letter.

$\therefore$ Total no. of ways $= 5 \times 1 \times 1 = 5$

Hence, sum of all the possible no. of ways in which the string of letters can be formed = 20 + 10 + 5 = 35.

**10.**

| 1st Letter | 2nd Letter | 3rd Letter | No. of ways |
|---|---|---|---|
| $a, i, o, u$ | $m$ | $e$ | $4 \times 1 \times 1 = 4$ |
| $a, e, i, o, u$ | $n$ | $e$ | $5 \times 1 \times 1 = 5$ |
| $e$ | $p$ | $e$ | $1 \times 1 \times 1 = 1$ |
| | | Total no. of ways = 10 | |

**11.** If $4^1$ is divided by 6, remainder is 4
If $4^2$ is divided by 6, remainder is 4
If $4^3$ is divided by 6, remainder is 4
If $4^4$ is divided by 6, remainder is 4
If $4^5$ is divided by 6, remainder is 4
… … … …
… … … …
If $4^{96}$ is divided by 6, remainder is 4

**12.** $$5x + 19y = 64 \quad \ldots(1)$$

$$\Rightarrow \quad x = \frac{64 - 19y}{5}$$

Following tables shows some values of $x$ and $y$ which satisfy the given equation (1).

(A)

| $y$ | $x$ |
|---|---|
| 1 | 9 |
| 6 | – 10 |
| 11 | – 29 |
| 16 | – 48 |
| 21 | – 67 |
| 26 | – 86 |
| 31 | – 105 |

(B)

| $y$ | $x$ |
|---|---|
| – 4 | 28 |
| – 9 | 47 |
| – 14 | 66 |
| – 19 | 85 |
| – 24 | 104 |
| – 29 | 123 |
| – 34 | 142 |

Choice (a) is wrong since for $x < 300$ and $y < 0$, there exists various solutions.
Choice (b) is wrong since for $x > 250$ and $y > -100$ there exists various solutions *e.g.*, for $x = 256$, $y = -64$ and $x = 275$, $y = -69$ etc.
Choice (d) is wrong since at $y = -58$ and $y = -57$ there is no solution (see the patter for $y$ is table B).
Choice (c) is correct since there exists a solution for $x = 256, 275$ etc.

**13.** LHS $= \frac{1}{3} \log_3 M + 3 \log_3 N$

$= \log_3 M^{1/3} + \log_3 N^3$

$= \log_3 (M^{1/3} N^3)$

RHS $= 1 + \log_{0.008} 5$

$= 1 + \log_{5^{-3}} 5 \qquad \left(\because \ 0.008 = \frac{8}{1000} = \frac{1}{125} = 5^{-3}\right)$

$= 1 + \left(\frac{\log_5 5}{\log_5 5^{-3}}\right)$

$= 1 + \frac{1}{-3}\left(\frac{\log_5 5}{\log_5 5}\right)$

$= 1 + \left(-\frac{1}{3}\right) = \frac{2}{3}$

$\because \quad$ LHS = RHS

$\therefore \quad \log_3 (M^{1/3} N^3) = \frac{2}{3}$

$\Rightarrow \quad M^{1/3} N^3 = 3^{2/3}$ (cubing both the sides)

$\Rightarrow$ Value of coins / $\Rightarrow$ Amount required

$MN^9 = 9$

$N^9 = \frac{9}{M}$

**14.** In order to require minimum no. of coins we need to have the coins of greater denominations.

| | No. of Coins of | | | | | Total No. of Coins |
|---|---|---|---|---|---|---|
| | 50 paise | 25 paise | 10 paise | 5 paise | 2 paise | |
| 78 paise | 1 | — | 2 | — | 4 | 7 |
| 69 paise | 1 | — | 1 | 1 | 2 | 5 |
| 101 paise | 1 | 1 | 2 | — | 3 | 7 |
| | | | | | Total | 19 |

Thus, the required no. of coins = 19.

**15.** For the given perimeter, the polygon with maximum no. of sides has maximum area.
Hence $c > s > t$.

**NOTE** A circle has infinite no. of sides.

**Alternatively :** Let the circumference of the circle, perimeter of equilateral triangle and perimeter of square be same and equal to 132 cm (132 = LCM of 22, 4, 3)

$\therefore \quad$ Radius of circle = 21 cm

Each side of $\Delta$ = 44 cm

and $\quad$ each side of square = 33 cm

$\therefore \quad \text{Area of circle} = \pi r^2 = \frac{22}{7} \times (21)^2 = 1386 \text{ cm}^2$

$\text{Area of triangle} = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4} \times (44)^2$

$= 484\sqrt{3} = 838.288 \text{ cm}^2$

and $\text{Area of square} = a^2 = (33)^2 = 1089$

$\therefore \quad c > s > t$

**Alternatively :** Let circumference of the circle

= perimeter of triangle

= perimeter of square

$= P$

Then $\quad$ radius of circle $= \frac{P}{2\pi}$

each side of $\Delta = \frac{P}{3}$

each side of Square $= \frac{P}{4}$

$\therefore \quad \text{area of circle} = \pi r^2 = \pi \times \left(\frac{P}{2\pi}\right)^2$

$= \frac{P^2 \times 7}{4 \times 22} = \frac{7}{88} P^2$

$\text{area of triangle} = \frac{\sqrt{3}}{4} a^2 = \frac{\sqrt{3}}{4}\left(\frac{P}{3}\right)^2 = \frac{P^2}{12\sqrt{3}}$

and area of square $= a^2 = \left(\frac{P}{4}\right)^2 = \frac{P^2}{16}$

Since $\frac{7}{88} > \frac{1}{16} > \frac{1}{12\sqrt{3}}$

$\therefore \quad c > s > t$

**16.** $MDCCLXXXVII = 1000 + 500 + 100 + 100 + 50 + 10 + 10 + 10 + 5 + 1 + 1$

$= 1787$

**17.** $MCMXCIX = 1000 + (1000 - 100) + (100 - 10) + (10 - 1)$

$= 1999$

**18.** (a) $MCMLXXV = 1000 + (1000 - 100) + 50 + 10 + 10 + 5$

$= 1975$

(b) $MCMXCV = 1000 + (1000 - 100) + (100 - 10) + 5$

$= 1995$

(c) $MVD = 1000 + (500 - 5) = 1495$

(d) $MVM = 1000 + (1000 - 5) = 1995$

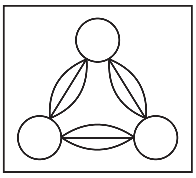
Hence, (c) is the right choice.

**19.** Since all the 12 towns are connected with each other with at least one direct line.

Total no. of single direct lines among 12 towns $= {}^{12}C_2 = 66$

But the towns which belong to the same zone are connected by 3 direct lines. So we need to connect 3 towns of each zone with 2 more direct lines (since they are already connected with 1 single direct line)

$\therefore$ No. of additional lines in one zone $= {}^{3}C_2 \times 2 = 6$

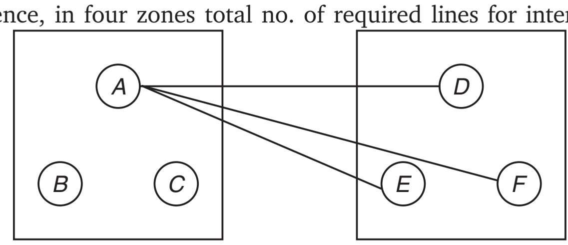


$\therefore$ Total no. of additional lines in four zones $= 6 \times 4 = 24$

$\therefore$ Required number of direct telephone lines $= 66 + 24 = 90$

**Alternatively :** Consider a particular zone having 3 towns. Each town is directly connected with 3 lines inside the same zone. Therefore total number of lines required for one zone $= {}^{3}C_2 \times 3 = 9$

Hence, in four zones total no. of required lines for internal connections $= 9 \times 4 = 36$




Each town in first zone can be connected to 3 towns in the second zone. Therefore between two particular zones total no. of required lines $= 3 \times 3 = 9$

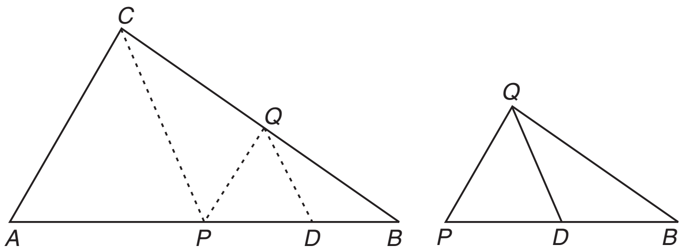
[$AD$, $AE$, $AF$; $BD$, $BE$, $BF$; $CD$, $CE$, $CF$]

Therefore total no. of lines required for connecting towns of different zones $= {}^{4}C_2 \times 9 = 6 \times 9 = 54$

(Since out of 4 zones any two zones can be selected in ${}^{4}C_2$ ways.)

Hence, the total no. of lines $= 36 + 54 = 90$

**20.**




Given that $AP : PB = 4x : 3x$

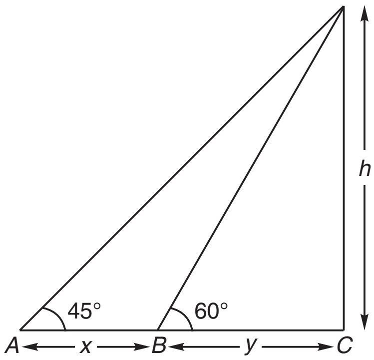
Since $\Delta ABC \sim \Delta PBQ$ ($AC \parallel PQ$, $\angle B$ is common)

$\therefore \quad PD : DB = 4y : 3y$ ($\because DQ \parallel PC$)

(As $P$ divides $AB$ in the ratio $4 : 3$ so $D$ divides $PB$ in the same ratio $4 : 3$)

But $\quad PB = PD + DB$

$\Rightarrow \quad 3x = 4y + 3y = 7y$




$\Rightarrow \quad \frac{x}{y} = \frac{7}{3}$

we are required to find $\frac{AP}{PD} = \frac{4x}{4y} = \frac{x}{y} = \frac{7}{3}$

**21.** Let $TC$ be the tower. Initially car is at $A$ where car makes 45° angle and it reaches $B$, where it makes 60° with the top of the tower.

Let $\quad AB = x, \quad BC = y \quad$ and $\quad TC = h$

In $\Delta ATC$, $\quad \tan 45° = \frac{h}{x + y}$

$\Rightarrow \quad 1 = \frac{h}{x + y}$

$\Rightarrow \quad h = x + y \quad$ ...(1)

In $\Delta BTC$, $\quad \tan 60° = \frac{h}{y}$

$\Rightarrow \quad \sqrt{3} = \frac{h}{y}$

$\Rightarrow \quad h = y\sqrt{3} \quad$ ...(2)

$\therefore$ From (1) and (2)

$y\sqrt{3} = x + y$

$\Rightarrow \quad \frac{y}{x} = \frac{1}{(\sqrt{3} - 1)}$

Since the speed of car is constant, therefore the time taken by car is directly proportional to the distance covered.

$\therefore \quad \frac{y}{x} = \frac{T_y}{T_x} = \frac{1}{(\sqrt{3} - 1)} \quad$ ($T \to$ time)

$\because$ To cover $x$ m distance car takes 10 min

$\therefore$ To cover $y$ m distance car will take $10 \times \dfrac{1}{(\sqrt{3}-1)}$ min

$$= \frac{10}{(\sqrt{3}-1)} \times \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$$

$$= 5(\sqrt{3}+1) \text{ min}$$

**22.** Let $\angle CAD = \angle ACD = x$ $\quad (\because AD = CD)$

$\therefore \quad \angle CDB = x + x = 2x$ $\quad$ (exterior angle)

$\therefore \quad \angle CBD = \angle CDB = 2x$ $\quad (\because CD = BC)$

Let $\quad \angle DCB = y$

$\therefore \quad \angle ACB = \angle ACD + \angle DCB = x + y = 84°$ $\quad \ldots(1)$

and in $\Delta DCB$, $\angle CDB + \angle DBC + \angle DCB = 180°$ $\quad \ldots(2)$

$\Rightarrow \quad 2x + 2y + y = 4x + y = 180°$ $\quad \ldots(2)$

from equations (1) and (2), we get,

$$y = 52° \quad \text{and} \quad x = 32°$$

$\therefore \quad \angle DBC = 2x = 64°$

**23.** $ax^2 + bx + c = 0$ has real roots if $b^2 - 4ac \ge 0$

$\therefore$ $ax^2 + bx + 1 = 0$ will have real roots if $b^2 - 4ac \ge 0$

or $\quad b^2 \ge 4a$ $\quad$ (here $c = 1$)

| Value of $a$ | Corresponding Value of $b$ for $b^2 \ge 4a$ | No. of Ways |
|---|---|---|
| 1 | 2, 3, 4 | 3 |
| 2 | 3, 4 | 2 |
| 3 | 4 | 1 |
| 4 | 4 | 1 |
| | Total → | 7 |

Hence total no. of ways $= 3 + 2 + 1 + 1 = 7$

**24.** $$\log_{10} x - \log_{10} \sqrt{x} = 2 \log_x 10$$

$$\Rightarrow \quad \log_{10} x - \frac{1}{2} \log_{10} x = \frac{2}{\log_{10} x}$$

$$\Rightarrow \quad \frac{1}{2} \log_{10} x = 2 \cdot \frac{1}{\log_{10} x}$$

$$\Rightarrow \quad (\log_{10} x)^2 = 4$$

$$\Rightarrow \quad \log_{10} x = \pm 2$$

$$\Rightarrow \quad x = 10^{-2} \text{ or } 10^2$$

$$\Rightarrow \quad x = \frac{1}{100} \text{ or } 100$$

Hence (b) is the correct option.

**25.** The required two digit number is of the form $7n + 3$. These numbers are as follows :

$$10, 17, 24, 31, 38, \ldots 94$$

The first number is obtained by substituting $n = 1$ and last number is obtained by substituting $n = 13$, hence there are 13 such numbers.

$\therefore$ Sum of these 13 numbers $= 10 + 17 + \ldots 94$

$$= \left(\frac{10 + 94}{2}\right) \times 13 \qquad \left\{\because S_n = \left(\frac{a + l}{2}\right) n\right\}$$

$$= 676$$

**26.** $\angle BAC = \angle ACT + \angle ATC = 50° + 30° = 80°$

and $\quad \angle ACT = \angle ABC$ $\quad$ (angle in alternate segment)

$\therefore \quad \angle ABC = 50°$

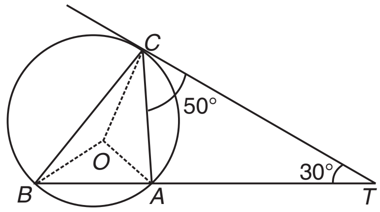
Hence, $\quad \angle BCA = 180° - (\angle ABC + \angle BAC)$

$$= 180° - (50° + 80°) = 50°$$

Now, since $\angle BOA = 2(\angle BCA)$ $\quad$ (using theorem)

$\therefore \quad \angle BOA = 100°$

**Alternatively :**




$$\angle CAB = 50° + 30° = 80°$$

$(\because \angle CAB$ is an exterior angle of $\Delta CTA)$

$\therefore \quad \angle COB = 2 \times 80° = 160°$

$(\because \angle COB$ is the central angle and twice of $\angle CAB)$

Again $\quad \angle OCT = 90°$ $\quad (\because CT$ is a tangent$)$

$\therefore \quad \angle OCA = 40°$

$\therefore \quad \angle OAC = \angle OCA = 40°$ $\quad (\because OA = OC)$

$\therefore \quad \angle AOC = 180 - (\angle OAC + \angle OCA)$

$\Rightarrow \quad \angle AOC = 100°$

$\therefore \quad \angle BOA + \angle BOC + \angle AOC = 360°$

$\Rightarrow \quad \angle BOA = 360 - (160 + 100)$

$\Rightarrow \quad \angle BOA = 100°$

**Alternatively :** $\angle OCA = \angle OAC = 40°$

$\therefore \quad \angle OAB = \angle CAB - \angle OAC$

$$= 80° - 40° = 40°$$

$\therefore \quad \angle OBA = \angle OAB = 40°$

$\therefore \quad \angle BOA = 180° - (40° + 40°)$

$(\because \angle BOA + \angle OBA + \angle OAB = 180°)$

$\Rightarrow \quad \angle BOA = 100°$

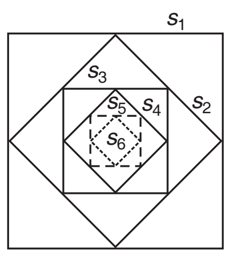
**27.** $\log m + \log\left(\dfrac{m^2}{n}\right) + \log\left(\dfrac{m^3}{n^2}\right) + \log\left(\dfrac{m^4}{n^3}\right) + \ldots n^{\text{th}}$ term

$$= \log\left[\frac{m \cdot m^2 \cdot m^3 \cdot m^4 \ldots m^n}{n \cdot n^2 \cdot n^3 \cdot n^4 \ldots n^{n-1}}\right]$$

$$= \log\left[\frac{m^{(1+2+3+\ldots n)}}{n^{(1+2+3+\ldots(n-1))}}\right]$$

$$= \log\left[\frac{m^{\frac{n(n+1)}{2}}}{n^{\frac{(n-1)n}{2}}}\right]$$

$$= \log\left[\frac{m^{(n+1)}}{n^{(n-1)}}\right]^{n/2}$$




**28.** Let each side of $S_1$ be $a_1$.

Now, since each next square *i.e.*, $S_2, S_3, S_4 \ldots$ etc. is formed by joining the mid points of the sides of its previous square *i.e.*, $S_1, S_2, S_3 \ldots$ etc.

Hence the each side of the next square will be $\frac{1}{\sqrt{2}}$ times the side of the previous square, respectively

*e.g.*, Side of $S_2 = \frac{1}{\sqrt{2}}$ (side of $S_1$)

Side of $S_3 = \frac{1}{\sqrt{2}}$ (side of $S_2$)

Side of $S_4 = \frac{1}{\sqrt{2}}$ (side of $S_3$) etc.

Thus if $a_1, a_2, a_3 \ldots a_n$ be the sides of the squares $S_1, S_2, S_3, \ldots S_n$ respectively

Then $a_1 = a$ (say)

$\therefore \quad a_2 = \frac{a}{\sqrt{2}}$

$a_3 = \frac{a}{2}$

$a_4 = \frac{a}{2\sqrt{2}}$ etc.

$\therefore \quad P_1 = 4a,\ P_2 = 2\sqrt{2}a,\ P_3 = 2a,\ P_4 = \sqrt{2}a, \ldots$

and $A_1 = a^2,\ A_2 = \frac{a^2}{2},\ A_3 = \frac{a^2}{4},\ A_4 = \frac{a^2}{8}, \ldots$

$$\therefore \quad P_1 + P_2 + P_3 + \ldots = a(4 + 2\sqrt{2} + 2 + \sqrt{2} + \ldots \infty)$$

$$= a\left(\frac{4}{1 - \frac{1}{\sqrt{2}}}\right) = \frac{a \times 4\sqrt{2}}{(\sqrt{2}-1)}$$

$$\text{and} \quad A_1 + A_2 + A_3 + \ldots = a^2\left(1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \ldots \infty\right)$$

$$= a^2\left(\frac{1}{1 - \frac{1}{2}}\right) = 2a^2$$

$$\therefore \quad \frac{P_1 + P_2 + P_3 + \ldots}{A_1 + A_2 + A_3 + \ldots} = \frac{\left(\frac{a \times 4\sqrt{2}}{(\sqrt{2}-1)}\right)}{2a^2}$$

$$= \frac{2\sqrt{2}}{(\sqrt{2}-1)a} = \frac{2\sqrt{2}(\sqrt{2}+1)}{a}$$

$$= \frac{2(2+\sqrt{2})}{a}$$

**29.** Since $y - x = z - y \therefore x, y$ and $z$ are in AP

Let '$d$' be the common difference of the AP

Therefore $x = y - d$ and $z = y + d$

$\therefore \quad xyz = 4$

$\Rightarrow \quad (y-d)y(y+d) = 4$

$\Rightarrow \quad y(y^2 - d^2) = 4$

$y$ will be minimum when $y^2 - d^2$ will be maximum and $y^2 - d^2$ will be maximum when $d^2$ is 0.

$\therefore \quad y(y^2 - d^2) = 4$

$\Rightarrow \quad y(y^2 - 0) = 4$

$\Rightarrow \quad y^3 = 4 = 2^2$

$\Rightarrow y = 2^{2/3}$ (taking cube root of both sides)

**30.** Total number of two digit numbers having distinct digits $= 9 \times 9 = 81$

There are four digits which create confusion (except to 0) : 1, 6, 8, 9.

From these four digits we can form two digit codes which can create confusion $= 4 \times 3 = 12$ ($\because$ digits are distinct)

But out of these 12 numbers 2 numbers are 69 and 96 which do not create confusion.

$\therefore$ Total no. of two digit codes which create confusion

$= 12 - 2 = 10$

Hence, the required no. of codes $= 81 - 10 = 71$

**31.** Let the side of original square be '$a$', then the area of square $= a^2$

and area of largest possible circle in the square

$$= \pi\left(\frac{a}{2}\right)^2 = \frac{\pi a^2}{4}$$

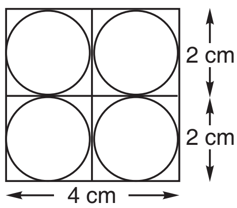
$\therefore$ area left after cutting out the circle from square

$$= a^2 - \frac{\pi a^2}{4} = a^2\left(1 - \frac{\pi}{4}\right)$$

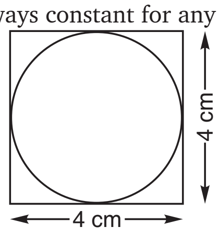
$$\frac{\text{Area of scrapped cloth}}{\text{Area of original cloth}} = \frac{a^2\left(1 - \frac{\pi}{4}\right)}{a^2}$$

$$= \left(1 - \frac{\pi}{4}\right),$$

which is a constant.




Since, the ratio of remaining part and original square is constant. Therefore we can infer that the percentage area of scrapped cloth is always constant for any size of square shaped cloth.



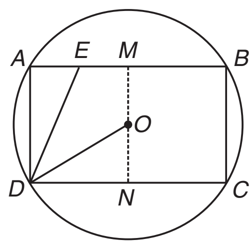

Thus, we can say that in each case (*i.e.*, first process and second process) the area of scrapped cloth is same since the area of the original piece of cloth is same in each case.

**Alternatively :** Since it is given that "the original square of side $a$ is divided into $n$ smaller squares, **not necessarily of the same size.**" Therefore we can assume that (for convenience) each smaller square is of the same size.

Let the each side of larger (*i.e.*, original) square be '$a$' = 4 cm and assume that it is being divided into four smaller squares of side 2 cm each.

**Process I :** Area of each of the smaller square $= (2)^2 = 4\ \text{cm}^2$

Area of maximum possible circle in each of the square $= \pi(1)^2 = \pi\ \text{cm}^2$




$\therefore$ Area of scrapped (remaining) cloth $= (4 - \pi)\ \text{cm}^2$

Hence, total area of scrapped cloth = $4(4-\pi)\ \text{cm}^2$

**Process II :** Area of the original square = $(4)^2 = 16\ \text{cm}^2$

Area of largest possible circle inside the square = $\pi \times (2)^2 = 4\pi$

$\therefore$ Area of scrapped cloth

$$= 16 - 4\pi = 4(4-\pi)\ \text{cm}^2$$

$\therefore$ Ratio of the scrapped cloth in both the process

$$= \frac{4(4-\pi)}{4(4-\pi)} = \frac{1}{1}$$

**NOTE** This can also be proved algebrically (*i.e.*, by taking variables) instead of assuming some value of *a* and *n* as above.

**32.** Recall that the diagonal of an inscribed rectangle bisect each other at the centre of the circle.
Now, join the mid points of $AB$ and $CD$ (*i. e.*, $M$ and $N$) passing through $O$, the centre of the circle $MN$ is parallel to $AD$ and $BC$ both, where $OM = ON$.
Triangles $EAD$ and $OND$ are similar

where, $$\frac{EA}{ON} = \frac{AD}{ND} = \frac{ED}{OD}$$

**Now go through options :** Let us consider option (a)
As from option (a)

$$\frac{AE}{AD} = \frac{1}{\sqrt{3}} = \frac{x}{\sqrt{3}x}$$

$\therefore$ $$\frac{ON}{DN} = \frac{x}{\sqrt{3}x} \qquad \left(\because \frac{AE}{AD} = \frac{ON}{DN}\right)$$

$\therefore$ $OD = 2x$ (radius of the circle)

(Using Pythagorus theorem)

$\therefore$ Area of circle = $\pi \times (2x)^2 = 4x^2\pi$

Now, length of rectangle = $CD = 2DN = 2\sqrt{3}x$

and breadth of rectangle = $AD = MN = 2(ON) = 2x$

$\therefore$ Area of rectangle = $(2\sqrt{3}x) \times (2x) = 4\sqrt{3}x^2$

$\therefore$ $$\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{4x^2\pi}{4\sqrt{3}x^2} = \frac{\pi}{\sqrt{3}}$$

Since, the given condition (that the ratio of area's of circle to the rectangle is $\frac{\pi}{\sqrt{3}}$) is satisfied, hence the assumed option (a) is correct.

**Alternatively :** Since, $\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\pi}{\sqrt{3}}$

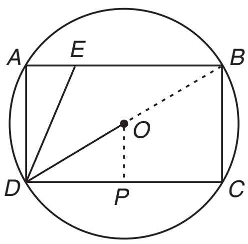
$$\frac{\pi (OD)^2}{CD \times BC} = \frac{\pi}{\sqrt{3}}$$

$\Rightarrow$ $$\frac{OD^2}{CD \times BC} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$\frac{DN^2 + ON^2}{(2DN)\times(2ON)} = \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$\frac{DN^2 + ON^2}{DN \times ON} = \frac{4}{\sqrt{3}}$$

$\Rightarrow$ $$\frac{DN}{ON} + \frac{ON}{DN} = \frac{4}{\sqrt{3}}$$




$\Rightarrow$ $$\frac{1}{\frac{ON}{DN}} + \frac{ON}{DN} = \frac{4}{\sqrt{3}}$$

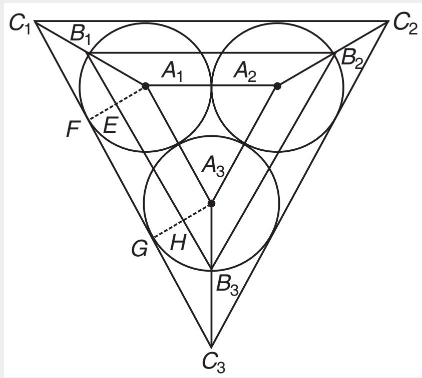
Let $$\frac{ON}{DN} = x$$

$\therefore$ $$\frac{1}{x} + x = \frac{4}{\sqrt{3}} = \sqrt{3} + \frac{1}{\sqrt{3}}$$

$\Rightarrow$ $$x = \frac{1}{\sqrt{3}} = \frac{ON}{DN} = \frac{AE}{AD} \qquad (\because \Delta DNO \sim \Delta DAE)$$

$\therefore$ $$\frac{AE}{AD} = \frac{1}{\sqrt{3}}$$

**Alternatively :** Produce $DO$ to $B$ and drop a perpendicular from $O$ on $CD$.




$\therefore$ $\Delta ODP \sim \Delta BDC$

Now, go through options, Let $\frac{AE}{AD} = \frac{1}{\sqrt{3}}$

$\therefore$ $$\frac{BC}{CD} = \frac{OP}{DP} = \frac{AE}{AD} = \frac{1}{\sqrt{3}} = \frac{x}{\sqrt{3}x}$$

$\therefore$ Area of circle = $\pi \times (OD)^2 = \pi \times \left(\frac{BD}{2}\right)^2$

$$= \pi \times (x)^2 \qquad (\because BD = \sqrt{x^2 + (\sqrt{3}x)^2})$$

and Area of rectangle = $BC \times CD$

$$= x \times \sqrt{3}x = \sqrt{3}x^2$$

$\therefore$ $$\frac{\text{Area of circle}}{\text{Area of rectangle}} = \frac{\pi x^2}{\sqrt{3}x^2} = \frac{\pi}{\sqrt{3}},$$

Hence chosen option (a) is correct.

✍ ***Solutions for question no. 33, 34, and 35 :*** *Let the radius of each circular park be 'r'.'*

$\therefore$ $A_1A_2 = A_2A_3 = A_3A_1 = 2r$

In $\Delta A_1B_1E$, $\angle A_1B_1E = 30°$

$\therefore$ $$\cos 30° = \frac{B_1E}{B_1A_1} = \frac{B_1E}{r}$$

$\Rightarrow$ $$\frac{\sqrt{3}}{2} = \frac{B_1E}{r}$$

$\Rightarrow$ $$B_1E = \frac{r\sqrt{3}}{2} = B_3H$$

$\therefore$ $$B_1B_3 = B_1E + EH + HB_3$$
$$= \frac{r\sqrt{3}}{2} + 2r + \frac{r\sqrt{3}}{2}$$

$\Rightarrow \quad B_1B_3 = r(\sqrt{3}+2)$

$\therefore \quad B_1B_2 = B_2B_3 = B_1B_3 = r(\sqrt{3}+2)$

*Again, In* $\Delta A_1C_1F$, $\angle A_1C_1F = 30°$

$\therefore \quad \tan 30° = \dfrac{A_1F}{C_1F}$

$$\frac{1}{\sqrt{3}} = \frac{r}{C_1F}$$

$\Rightarrow \quad C_1F = r\sqrt{3} = GC_3$

$\therefore \; C_1C_3 = C_1F + FG + GC_3 = r\sqrt{3} + 2r + r\sqrt{3}$

$\Rightarrow \; C_1C_3 = r(2\sqrt{3}+2)$

$\therefore \; C_1C_2 = C_2C_3 = C_1C_3 = r(2\sqrt{3}+2)$

**33.** $a = A_1A_2 + A_2A_3 + A_3A_1 = 3(2r) = 6r$

$b = B_1B_2 + B_2B_3 + B_3B_1 = 3[r(\sqrt{3}+1)] = 3r(\sqrt{3}+2)$

$c = C_1C_2 + C_2C_3 + C_3C_1 = 3[r(2\sqrt{3}+1)] = 3r(2\sqrt{3}+2)$

$\therefore \quad b - a = c - b = 3\sqrt{3}r$

**34.** Time taken by $A = \dfrac{2r}{20} + \dfrac{2r}{30} + \dfrac{2r}{15}$

$= \dfrac{3r}{10}$ sec (say)

Distance traversed by $B$ in $\dfrac{3r}{10}$ sec $= \dfrac{3r}{10} \times (10\sqrt{3}+20)$

$= 3r(\sqrt{3}+2)$

Since $3r(\sqrt{3}+2)$ is the distance of the complete path. Hence $B$ reaches again at $B_1$.

Time taken by $C$ to traverse $C_1C_2 = \dfrac{r(2\sqrt{3}+2)}{\frac{40}{3}(\sqrt{3}+1)} = \dfrac{3r}{20}$

Similarly, time taken by $C$ to traverse $C_2C_3 = \dfrac{3r}{20}$

Hence $C$ covers $C_1C_2 + C_2C_3$ distance in

$$\frac{3r}{20} + \frac{3r}{20} = \frac{3r}{10} \text{ sec}$$

Thus $C$ reaches at $C_3$ in $\dfrac{3r}{10}$ sec

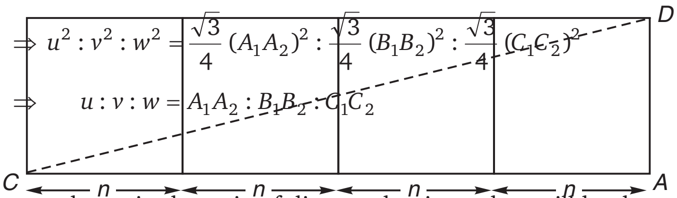
Thus when $A$ reaches $A_1$, $B$ also reaches $B_1$ and $C$ reaches at $C_3$.

Hence, choice (c) is correct.

**35.** $u^2 : v^2 : w^2 =$ Area $A$ : Area $B$ : Area $C$

$\Rightarrow u^2 : v^2 : w^2 = \dfrac{\sqrt{3}}{4}(A_1A_2)^2 : \dfrac{\sqrt{3}}{4}(B_1B_2)^2 : \dfrac{\sqrt{3}}{4}(C_1C_2)^2$

$\Rightarrow \quad u : v : w = A_1A_2 : B_1B_2 : C_1C_2$



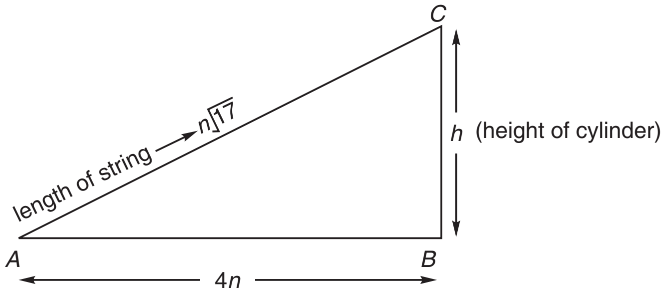

Since speeds are in the ratio of distace, the time taken will be the same by each sprinters.

Thus all of them will complete their sprint in same time *i. e.*, $A$, $B$, $C$ will reach at $A_3$, $B_3$, $C_3$ at the same time.

Hence, choice (b) is correct.

**36.** Vertical spacing between any two consecutive turns

$$= \frac{\text{height of cylinder}}{\text{no. of turns}} = \frac{h}{n}$$




**37.** If all the 4 successive (or consecutive) faces are opened up then the following figure will represent the position of string.

Then length of the string $= CD = \sqrt{(CA)^2 + (AD)^2}$

$= \sqrt{(4n)^2 + n^2} = n\sqrt{17}$

**38.** The string of length $n\sqrt{17}$ is wound on cylinder of height $h$ making $n$ rounds. Thus the total circumference in $n$ rounds

$= n \times$ actual circumference of cylinder

$= n \times 2\pi \left(\dfrac{2}{\pi}\right) = 4n$

If this phenomenon is represented in the geometrical form, we get the following figure.

$\therefore$ From the above figure,

$$(4n)^2 + (h)^2 = (n\sqrt{17})^2$$

$\Rightarrow \quad h = n$

**39.** Let $x = 2$ (a prime number) and

$y = 30$ (a composite number)

then choice (a) is wrong

since $y - x = 30 - 2 = 28$, which is an even integer.

Choice (b) is wrong

since $xy = 2 \times 30 = 60$, which is an even integer

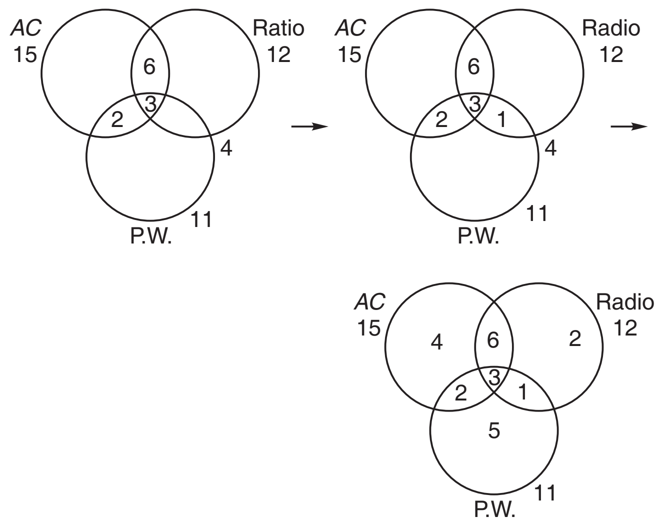
Choice (c) is also wrong.

since $\quad \dfrac{x+y}{x} = \dfrac{2+30}{2} = \dfrac{32}{2} = 16,$

which is an even integer

Hence, choice (d) is true.

**40.**



From the above venn-diagram it is clear that no. of cars which have atleast one option

$$= (4 + 2 + 5) + (6 + 2 + 1) + 3 = 23$$

Hence the no. of cars which have none of the options $= 25 - 23 = 2$

**41.** Let us substitute $n = 36$ in the given expression,

we get, $\frac{(36)^2 + 2\sqrt{36}(36 + 4) + 16}{36 + 4\sqrt{36} + 4} = \frac{1296 + 12(40) + 16}{64}$

$$= 28$$

Again if we put $n = 72$, then we get,

$$\frac{(72)^2 + 2\sqrt{72}\,(72 + 4) + 16}{72 + 4\sqrt{72} + 4} = 59 \text{ (approx.)}$$

Since $x = 28$, hence choice (d) is eliminated and for $x \approx 59$, choices (a) and (b) are eliminated.
Hence, choice (c) could be correct.

**42.** $13x + 1 < 2z$ and $z + 3 = 5y^2$

Combining the two equations, we get

$$13x + 7 < 10y^2$$

Let us consider $x = 1$ and $y = 2$, then we have

$$13 \times 1 + 7 < 10 \times (2)^2$$

$\Rightarrow$ $20 < 40$

when $(x = 1) < (y = 2)$, the given inequality is satisfied. Hence choice (b) and (c) are eliminated.
Now, we consider $x = 4$ and $y = 3$, we have

$$13 \times 4 + 7 < 10 \times (3)^2$$

$$59 < 90$$

Hence, choice (a) is also ruled out.
Therefore, choice (d) is best answer.

**43.** Any composite number $(n)$ which is not a perfect square has atleast one factor less than $\sqrt{n}$ and greater than $\sqrt{n}$, such that their product is $n$.

*e.g.*, $\sqrt{6} \approx 2.5$ then 6 has two factors 2 and 3; $(2 < 2.5 < 3)$

$\sqrt{8} \approx 2.85$ then 8 has two factors 2 and 4; $(2 < 2.85 < 4)$

$\sqrt{10} \approx 3.2$ then 10 has two factors 2, 5; $(2 < 3.2 < 3)$

The logic behind the question is that if a composite number is not a perfect square, then it must have at least two unequal factors (since non perfect square but composite) number can not have its square root as an integer unlikely in a perfect square number two factors are always equal. *e.g.*,

(i) $8 = 2 \times 4$

(ii) $10 = 2 \times 5$

(iii) $12 = 2 \times 6$ or $3 \times 4$

(iv) $15 = 3 \times 5$

(v) $18 = 2 \times 9$ or $3 \times 6$

See the (i) example.
$\sqrt{8} = \sqrt{2 \times 4} \approx 2.85 =$ GM of 2 and 4, which must lies between 2 and 4.
Similarly, in (ii) example
$\sqrt{10} = \sqrt{2 \times 5} \approx 3.2 =$ GM of 2 and 5, which must lies between 2 and 5.
Thus we can say that for a non perfect square composite number $\sqrt{n}$ is the geometric mean of its factors and the GM of any two or more numbers necessarily lies between them. So atleast one of the factor lies below $\sqrt{n}$ and other lies above $\sqrt{n}$.
Hence, choice (d) is correct.

**44.** $\because |b| \geq 1 \;\Rightarrow\; b \geq 1$ or $b \leq -1$.
Then we can chart out the following table.

| $a$ | $b$ | $x$ | $a - xb$ |
|---|---|---|---|
| $a > 0$ | $b \geq 1$ | –ve | $a - xb \geq 0$ |
| $a > 0$ | $b \leq -1$ | +ve | $a - xb \geq 0$ |
| $a < 0$ | $b \geq 1$ | –ve | $a - xb \geq 0$ |
| $a < 0$ | $b \leq -1$ | +ve | $a - xb \geq 0$ |

Hence, choice (b) is correct.
**Alternatively :** $|b| \geq 1$ and $x = -|a|\, b$

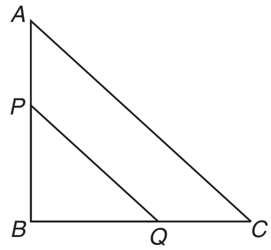
Now, $a - xb = a[-|a|\, b]\, b$

$\Rightarrow$ $a - xb = a + |a|\, b^2 \geq 0$

$\therefore$ $a - xb \geq 0$

$(\because |a|\, b^2$ is always positive and $|a|\, b^2 \geq a)$

**45.** $\frac{PQ}{AC} = \frac{65}{100}$




($PQ$ is 35% less than $AC$)

$\therefore \frac{\text{Area of } \Delta PBQ}{\text{Area of } \Delta ABC} = \frac{(PQ)^2}{(AC)^2}$

$$= \left(\frac{65}{100}\right)^2 = \frac{4225}{10000}$$

$\therefore$ Area of $\Delta PBQ = 0.4225 \times$ Area of $\Delta ABC$

$$= 0.4225 \times 34$$

$$= 14.365 \text{ sq. inch}$$

**46.** Let us consider there are 32 huts in the begining of the year 2001.

| Year | No. of Huts Rebuild | No. of Huts in the Begining of the Year | Huts Destroyed | Huts Remain-ing in the End of the Year |
|---|---|---|---|---|
| 2001 | — | 32 | 16 | 16 |
| 2002 | $2 \times 16 = 32$ | $32 + 16 = 48$ | 24 | 24 |
| 2003 | $2 \times 24 = 48$ | $48 + 24 = 72$ | 36 | 36 |
| 2004 | $2 \times 36 = 72$ | $72 + 36 = 108$ | 54 | 54 |

Now, go through the options.
Only option (c) satisfies the required condition.

*i.e.*, $54 < 24 + 36$

$\Rightarrow$ $54 < 60$

**47.** $a = 6b = 12c$ and $2b = 9d = 12e$

$\Rightarrow$ $a : b : c = 1 : \frac{1}{6} : \frac{1}{12}$

and $b : d : e = \frac{1}{2} : \frac{1}{9} : \frac{1}{12}$

$\Rightarrow$ $a : b : c = 12 : 2 : 1$

or $b : d : e = 18 : 4 : 3$

$\therefore$ $a : b : c : d : e = 108 : 18 : 9 : 4 : 3$

or $a = 108k$, $b = 18k$, $c = 9k$, $d = 4k$ and $e = 3k$; $k \in I$

Now, go throug options :

Choice (a) is wrong, since $\left(\frac{a}{27}, \frac{b}{e}\right)$ gives integer.

*i.e.*, $\left(\frac{108k}{27}, \frac{18k}{3k}\right) \Rightarrow (4k, 6k)$

Similarly, choice (b) and (c) are also eliminated.
choice (d) is correct

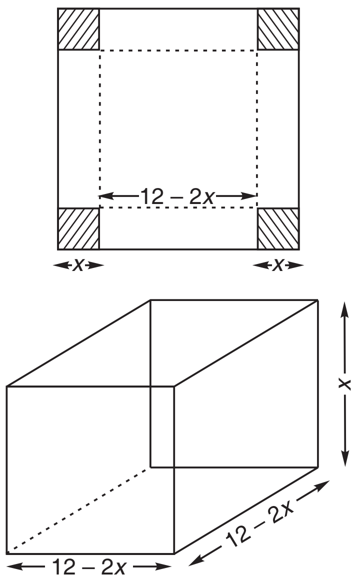
As, $\left(\frac{a}{b}, \frac{c}{d}\right) \Rightarrow \left(\frac{108k}{6}, \frac{9k}{4k}\right)$

(Since $\frac{9}{4}$ is not an integer)

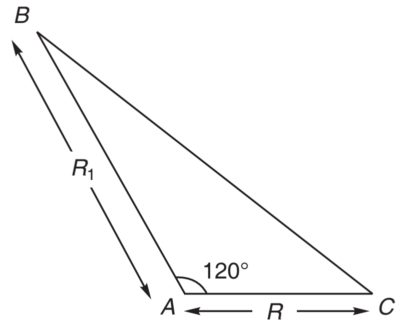
**48.** There is only one possible set of prime numbers *viz.*, 3, 5, 7. Hence, choice (a) is correct.
Here you can see some other sets where '$a$' is a prime number but either $(a + 2)$ or $(a + 4)$ is not a prime number :

(5, 7, 9 ); (7, 9 , 11); (11, 13, 15 ); (13, 15 , 17); (17, 19, 21 ); (19, 21 , 23); (23, 25 , 27); (29, 31, 33 ); (31, 33 , 35); (37, 39 , 41); etc

**49.**




Since the square of side $x$ inch is being removed from each corner, the length and breadth of the base of the sheet (*i.e.*, the base of the cuboid) reduces to $(12 - 2x)$ inch. Therefore the area of the base $= (12 - 2x)^2$ and thus the height of the cuboid becomes $x$ inch.




$\therefore$ the volume of the cuboid = base area × height

$= (12 - 2x)^2 \times x$

Now, going through options we find that at $x = 2$, the volume of the cuboid maximizes.
Hence, choice (d) is correct.

**50.** Since, the speed of Ram is 3 km/h. Hence the distance between $A$ and $B$ is $3 \times 4 = 12$ km.
Again since the speed of Shyam is 2 km/h. Hence the distance between $A$ and $C$ is $2 \times 4 = 8$ km.

$\therefore$ $AB = 12$ km and $AC = 8$ km

Using cosine rule, we get

$$\cos A = \frac{AB^2 + AC^2 - BC^2}{2 \cdot AB \cdot AC}$$

$$\cos 120° = \frac{144 + 64 - BC^2}{2 \times 12 \times 8}$$

$$\Rightarrow \left(-\frac{1}{2}\right) = \frac{208 - BC^2}{192}$$

$$\Rightarrow BC^2 = 304 \text{ or } BC = 4\sqrt{19} \text{ km}$$

Time taken by Ram to return $= \frac{BC + AC}{3}$

$= \frac{4\sqrt{19} + 8}{3}$ hours

Time taken by Shyam to return $= \frac{BC + AC}{2}$

$= \frac{4\sqrt{19} + 12}{2}$ hours

$\therefore$ Time interval $= \frac{4\sqrt{19} + 12}{2} - \frac{4\sqrt{19} + 8}{3}$

$= \frac{2\sqrt{19} + 10}{3}$ hours

Hence, choice (b) is correct.

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