CAT SOLVED PAPER

Leaked Paper

No. of Questions : 50 *Time :* 40 *min*

NOTE Each wrong answer carry $\frac{1}{3}$rd negative mark.

Directions for Question no. 1 to 3 : *Answer the questions independently of each other.*

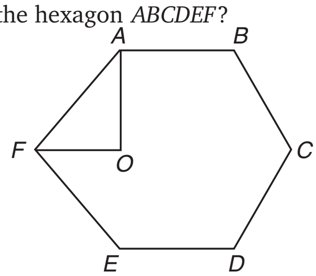
1. In the figure given below, *ABCDEF* is a regular hexagon and $\angle AOF = 90°$. *FO* is parallel to *ED*. What is the ratio of the area of the triangle *AOF* to that of the hexagon *ABCDEF*?

(a) $\frac{1}{12}$

(b) $\frac{1}{6}$

(c) $\frac{1}{24}$

(d) $\frac{1}{18}$



2. In a 4000 meter race around a circular stadium having a circumference of 1000 meters, the fastest runner and the slowest runner reach the same point at the end of the 5th minute, for the first time after the start of the race. All the runners have the same starting point and each runner maintains a uniform speed throughout the race. If the fastest runner runs at twice the speed of the slowest runner, What is the time taken by the fastest runner to finish the race ?

(a) 20 min (b) 15 min
(c) 10 min (d) 5 min

3. Given that $-1 \le v \le 1$, $-2 \le u \le -0.5$ and $-2 \le z \le -0.5$ and $w = \frac{vz}{u}$, then which of the following is necessarily true ?

(a) $-0.5 \le w \le 2$ (b) $-4 \le w \le 4$
(c) $-4 \le w \le 2$ (d) $-2 \le w \le -0.5$

Directions for Question no. 4 and 5 : *Answer the questions on the basis of the information given below :*
New age consultants have three consultants Gyani, Medha and Buddhi. The sum of the number of projects handled by Gyani and Buddhi individually is equal to the number of projects in which Medha is involved. All three consultants are involved together in 6 projects. Gyani works with Medha in 14 projects. Buddhi has 2 projects with Medha but without Gyani and 3 projects with Gyani but without Medha. The total number of projects for New age consultants is one less than twice the number of projects in which more than one consultant is involved.

4. What is the number of projects in which Gyani alone is involved ?

(a) Uniquely equal to zero
(b) Uniquely equal to 1
(c) Uniquely equal to 4
(d) Cannot be determined uniquely

5. What is the number of projects in which Medha alone is involved ?

(a) Uniquely equal to zero
(b) Uniquely equal to 1
(c) Uniquely equal to 4
(d) Cannot be determined uniquely

Directions for Question no. 6 to 8 : *Answer the questions on the basis of the information given below :*
A city has two perfectly circular and concentric ring roads, the outer ring road (OR) being twice as long as the inner ring road (IR). There are also four (straight line) chord roads from E_1, the east end point of OR to N_2, the north end point of IR, from N_1, the north end points of OR to W_2, the west end point of IR; from W_1, the west end point of OR, to S_2, the south end point of IR and from S_1, the south end point of OR to E_2, the east end point of IR. Traffic moves at a constant speed of 30π km/h. on the OR road, 20π km/hr on the IR road and $15\sqrt{5}$ km/h on all the chord roads.

6. The ratio of the sum of the lengths of all chord roads to the length of the outer ring road is :

(a) $\sqrt{5} : 2$ (b) $\sqrt{5} : 2\pi$
(c) $\sqrt{5} : \pi$ (d) none of these

7. Amit wants to reach N_2 from S_1. It would take him 90 minutes if he goes on minor arc $S_1 - E_1$ on OR, and then on the chord road $E_1 - N_2$. What is the radius of the outer ring road in kms?

(a) 60 (b) 40
(c) 30 (d) 20

8. Amit wants to reach E_2 from N_1 using first the chord $N_1 - W_2$ and then the inner ring road. What will be his travel time in minutes on the basis of information given in the above question ?

(a) 60 (b) 45
(c) 90 (d) 105

Directions for Question no. 9 to 18 : *Answer the questions independently of each other.*

9. When the curves $y = \log_{10} x$ and $y = x^{-1}$ are drawn in the $x - y$ plane, how many times do they intersect for values $x \ge 1$?

(a) Never (b) Once
(c) Twice (d) More than twice

10. The sum of 3rd and 15th elements of an arithmetic progression is equal to the sum of 6th, 11th and 13th elements of the same progression. Then which element of the series should necessarily be equal to zero ?

(a) 1st (b) 9th
(c) 12th (d) none of these

11. A test has 50 questions. A student scores 1 mark for a correct answer, $-\frac{1}{3}$ for a wrong answer, and $-\frac{1}{6}$ for not attempting a question. If the net score of a student is 32, the number of questions answered wrongly by that student cannot be less than :

(a) 6 (b) 12
(c) 3 (d) 9

12. How many even integers n, where $100 \le n \le 200$, are divisible neither by seven nor by nine ?

(a) 40 (b) 37
(c) 39 (d) 38

13. Which one of the following conditions must p, q and r satisfy so that the following system of linear simultaneous equations has at least one solution, such that $p + q + r \neq 0$?

$$x + 2y - 3z = p$$
$$2x + 6y - 11z = q$$
$$x - 2y + 7z = r$$

(a) $5p - 2q - r = 0$ (b) $5p + 2q + r = 0$
(c) $5p + 2q - r = 0$ (d) $5p - 2q + r = 0$

14. The function $f(x) = |x - 2| + |2.5 - x| + |3.6 - x|$, where x is a real number, attains a minimum at :

(a) $x = 2.3$ (b) $x = 2.5$
(c) $x = 2.7$ (d) none of these

15. Let A and B be two solid spheres such that the surface area of B is 300% higher than the surface area of A. The volume of A is found to be k% lower than the volume of B. The value of k must be :

(a) 85.5 (b) 92.5
(c) 90.5 (d) 87.5

16. Twenty-seven persons attend a party. Which one of the following statements can never be true ?

(a) There is a person in the party who is acquainted with all the twenty six others.
(b) Each person in the party has a different number of acquaintances.
(c) There is a person in the party who has an odd number of acquaintances.
(d) In the party, there is no set of three mutual acquaintances.

17. Let $g(x) = \max(5 - x, x + 2)$. The smallest possible value of $g(x)$ is :

(a) 4.0 (b) 4.5
(c) 1.5 (d) none of these

18. A positive whole number M is less than 100 is represented in base 2 notation, base 3 notation and base 5 notation. It is found that in all three cases the last digit is 1, while in exactly two out of the three cases the leading digit is 1. Then M equals :

(a) 31 (b) 63
(c) 75 (d) 91

✍ ***Directions for Questions no. 19 and 20 :*** *Answer the questions on the basis of the information given below :*

A certain perfume is available at a duty free shop at the Bangkok international airport. It is priced in the Thai currency Baht but other currencies are also acceptable. In particular, the shop accepts Euro and US dollar at the following rates of exchange :

US Dollar 1 = 41 Bahts

Euro 1 = 46 Bahts

The perfume is priced at 520 Bahts per bottle. After one bottle is purchased, subsequent bottles are available at a discount of 30%. Three friends S, R and M together purchase three bottles of the perfume, agreeing to share the cost equally. R pays 2 Euros, M pays 4 Euros and 27 Thai Bahts and S pays the remaining amount in US Dollars.

19. How much does R owe to S in Thai Baht ?

(a) 428 (b) 416
(c) 334 (d) 324

20. How much does M owe to S in US Dollars ?

(a) 3 (b) 4
(c) 5 (d) 6

✍ ***Directions for Questions 21 to 38 :*** *Answer the questions independently of each other.*

21. A leather factory produces two kinds of bags, standard and deluxe. The profit margin is Rs. 20 on a standard bag and Rs. 30 on a deluxe bag. Every bag must be processed on machine A and on machine B. The processing times per bag on the two machines are as follows :

	Time Required (Hours/bag)	
	Machine A	**Machine B**
Standard Bag	4	6
Deluxe Bag	5	10

The total time available on machine A is 700 hours and on machine B is 1250 hours. Among the following production plans, which one meets the machine availability constraints and maximizes the profit ?

(a) Standard 75 bags, deluxe 80 bags
(b) Standard 100 bags, deluxe 60 bags
(c) Standard 50 bags, deluxe 100 bags
(d) Standard 60 bags, deluxe 90 bags

22. At the end of year 1998, shepard bought nine dozen goats. Hence forth, every year he added p% of the goats at the beginning of the year and sold q% of the goats at the end of the year where $p > 0$ and $q > 0$. If shepard had nine dozen goats at the end of year 2002, after making the sales for the year, which of the following is true ?

(a) $p = q$ (b) $p < q$
(c) $p > q$ (d) $p = \frac{q}{2}$

23. Each side of a given polygon is parallel to either the X or the Y axis. A corner of such a polygon is said to be convex if the internal angle is 90° or concave if the internal angle is 270°. If the number of convex corners in such a polygon is 25, the number of concave corners must be :

(a) 20 (b) 0
(c) 21 (d) 22

24. Let p and q be the roots of the quadratic equation $x^2 - (\alpha - 2)x - \alpha - 1 = 0$. What is the minimum possible value of $p^2 + q^2$?

(a) 0 (b) 3
(c) 4 (d) 5

25. The 288th term of the series
$a, b, b, c, c, c, d, d, d, d, e, e, e, e, e, f, f, f, f, f, f \ldots$ is :

(a) u (b) v

(c) w (d) x

26. There are two concentric circles such that the area of the outer circle is four times the area of the inner circle. Let A, B and C be three distinct points on the perimeter of ths outer circle such that AB and AC are tangents to the inner circle. If the area of the outer circle is 12 square centimeters then the area (in square centimeters) of the triangle ABC would be :

(a) $\pi\sqrt{12}$ (b) $\frac{9}{\pi}$

(c) $\frac{9\sqrt{3}}{\pi}$ (d) $\frac{6\sqrt{3}}{\pi}$

27. Let a, b, c, d be four integers such that $a + b + c + d = 4m + 1$, where m is a positive integer. Given m, which one of the following is necessarily true ?

(a) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$

(b) The minimum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$

(c) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 - 2m + 1$

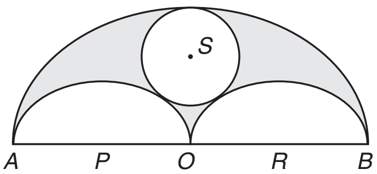
(d) The maximum possible value of $a^2 + b^2 + c^2 + d^2$ is $4m^2 + 2m + 1$

28. The number of non-negative real roots of $2^x - x - 1 = 0$ equals :

(a) 0 (b) 1

(c) 2 (d) 3

29. Three horses are grazing within a semi-circular field. In the diagram given below, AB is the diameter of the semi-circular field with centre at O. Horses are tied up at P, R and S such that PO and RO are the radii of semi circles with centres at P and R respectively, and S is the centre of the circle touching the two semi circles with diameters AO and OB. The horses tied at P and R can graze within the respective semi circles and the horses tied at S can graze within the circle centred at S. The percentage of the area of the semi circle with diameter AB that cannot be grazed by the horses is nearest to :

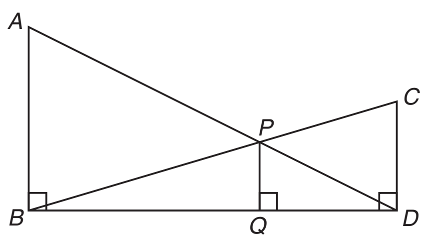


(a) 20 (b) 28

(c) 36 (d) 40

30. In a triangle ABC, $AB = 6$, $BC = 8$ and $AC = 10$. A perpendicular dropped from B, meets the side AC at D. A circle of radius BD (with centres B) is drawn. If the circle cuts AB and BC at P and Q respectively, then $AP : QC$ is equal to :

(a) 1 : 1 (b) 3 : 2



(c) 4 : 1 (d) 3 : 8

31. In the diagram given below, $\angle ABD = \angle CDB = \angle PQD = 90°$. If $AB : CD = 3 : 1$, the ratio of $CD : PQ$ is :

(a) 1 : 0.69 (b) 1 : 0.75

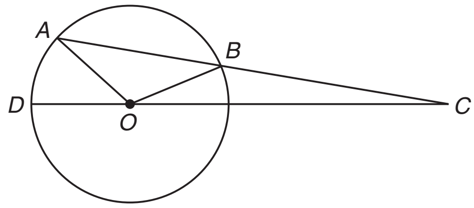
(c) 1 : 0.72 (d) none of these

32. If $\log_3 2$, $\log_3(2^x - 5)$, $\log_3\left(2^x - \frac{7}{2}\right)$ are in arithmetic progression, then the value of x is equal to :

(a) 5 (b) 4

(c) 2 (d) 3

33. In the figure given below, AB is the chord of a circle with centre O. AB is extended to C such that $BC = OB$. The straight line CO is produced to meet the circle at D. If $\angle ACD = y$ degrees and $\angle AOD = x$ degrees such that $x = ky$, then the value of k is :



(a) 3 (b) 2

(c) 1 (d) none of these

34. How many three digit positive integers, with digits x, y and z in the hundred's ten's, and unit's place respectively, exist such that $x < y$, $z < y$ and $x \neq 0$:

(a) 245 (b) 285

(c) 240 (d) 320

35. There are 8436 steel balls, each with a radius of 1 centimeter, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is :

(a) 34 (b) 38

(c) 36 (d) 32

36. If the product of n positive real numbers is unity, then their sum is necessarily :

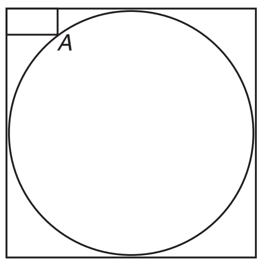
(a) a multiple of n

(b) equal to $n + \frac{1}{n}$

(c) never less than n

(d) a positive integer

37. In the figure below, the rectangle at the corner measures 10 cm × 20 cm. The corner A of the rectangle is also a point on the circumference of the circle. What is the radius of the circle in cm ?



(a) 10 cm (b) 40 cm

(c) 50 cm (d) none of the above

38. A vertical tower OP stands at the centre O of a square $ABCD$. Let h and b denote the length OP and AB respectively. Suppose $\angle APB = 60°$ the relationship between h and b can be expressed as :

(a) $2b^2 = h^2$ (b) $2h^2 = b^2$

(c) $3b^2 = 2h^2$ (d) $3h^2 = 2b^2$

✍ ***Directions for Questions no. 39 to 43:*** *Each question is followed by two statements, A and B. Answer each question using the following instructions.*

Choose 1. *If the question can be answered by one of the statements alone but not by the other.*

Choose 2. *If the question can be answered by using either statement alone.*

Choose 3. *If the question can be answered by using both the statements but cannot be answered by using either statement alone.*

Choose 4. *If the question cannot be answered even by using both the statements together.*

39. Is $a^{44} < b^{11}$, given $a = 2$ and b is an integer ?

A. b is even

B. b is greater than 16

40. What are unique values of b and c in the equation $4x^2 + bx + c = 0$ if one of the roots of the equation is $\left(-\frac{1}{2}\right)$?

A. The second root is $\frac{1}{2}$

B. The ratio of c and b is 1.

41. AB is a chord of a circle. $AB = 5$ cm. A tangent parallel to AB touches the minor arc AB at E. What is the radius of the circle?

A. AB is not a diameter of the circle.

B. The distance between AB and the tangent at E is 5 cm

42. If $\left(\frac{1}{a^2}+\frac{1}{a^4}+\frac{1}{a^6}+\ldots\right) > \left(\frac{1}{a}+\frac{1}{a^3}+\frac{1}{a^5}+\ldots\right)$?

A. $-3 \le a \le 3$

B. One of the roots of the equation $4x^2 - 4x + 1 = 0$ is 'a'.

43. D, E, F are the mid points of the sides AB, BC and CA of triangle ABC respectively. What is the area of DEF in square centimeters ?

A. $AD = 1$ cm, $DF = 1$ cm and perimeter of $DEF = 3$ cm

B. Perimeter of $ABC = 6$ cm, $AB = 2$ cm and $AC = 2$ cm

✍ ***Directions for Question no. 44 to 50 :*** *Answer the questions independently of each other.*

(a) 5 (b) 7
(c) 13 (d) 14

45. If x, y, z are distinct positive real numbers then $\frac{x^2(y+z)+y^2(x+z)+z^2(x+y)}{xyz}$ would be :

(a) greater than 4 (b) greater than 5
(c) greater than 6 (d) none of these

46. In a certain examination paper, there are n questions. For $J = 1, 2, \ldots n$, there are 2^{n-j} students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is :

(a) 12 (b) 11
(c) 10 (d) 9

47. Consider the following two curves in the $x-y$ plane : $y = x^3 + x^2 + 5$ and $y = x^2 + x + 5$

Which of the following statements is true for $-2 \le x \le 2$?

(a) The two curves intersect once
(b) The two curves intersect twice
(c) The two curves do not intersect
(d) The two curves intersect thrice

48. Let T be the set of integers $\{3, 11, 19, 27, \ldots 451, 459, 467\}$ and S be a subset of T such that the sum of no two elements of S is 470. The maximum possible number of elements in S is :

(a) 32 (b) 28
(c) 29 (d) 30

49. A graph may be defined as a set of points connected by lines called edges. Every edge connects a pair of points. Thus, a triangle is a graph with 3 edges and 3 points. The degree of a point is the number of edges connected to it. For example, a triangle is a graph with three points of degree 2 each. Consider a graph with 12 points. It is possible to reach any point from any other point through a sequence of edges. The number of edges, e in the graph must satisfy the condition :

(a) $11 \le e \le 66$ (b) $10 \le e \le 66$
(c) $11 \le e \le 65$ (d) $0 \le e \le 11$

50. There are 6 boxes numbered 1, 2, … 6. Each box is to be filled up either with a red or a green ball in such a way that at least 1 box contains a green ball and the boxes containing green balls are consecutively numbered. The total number of ways in which this can be done is :

(a) 5 (b) 21
(c) 33 (d) 60

Answers

1 (a)	**2.** (c)	**3.** (b)	**4.** (d)	**5.** (b)	**6.** (c)	**7.** (c)	**8.** (d)	**9.** (b)	**10.** (c)
11. (c)	**12.** (c)	**13.** (a)	**14.** (b)	**15.** (d)	**16.** (b)	**17.** (d)	**18.** (d)	**19.** (d)	**20.** (c)
21. (a)	**22.** (c)	**23.** (c)	**24.** (d)	**25.** (d)	**26.** (c)	**27.** (b)	**28.** (c)	**29.** (c)	**30.** (d)
31. (b)	**32.** (d)	**33.** (a)	**34.** (c)	**35.** (c)	**36.** (c)	**37.** (c)	**38.** (b)	**39.** (a)	**40.** (b)
41. (a)	**42.** (a)	**43.** (b)	**44.** (b)	**45.** (c)	**46.** (a)	**47.** (d)	**48.** (d)	**49.** (a)	**50.** (b)

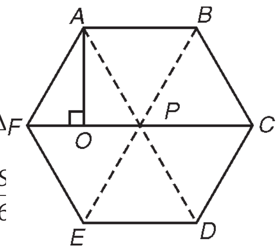
44. The number of positive integers n in the range $12 \le n \le 40$ such that the product $(n-1)(n-2)\ldots 3\cdot 2\cdot 1$ is not divisible by n is :

1. $\frac{\text{Area of } \Delta\, APF}{\text{Area of hexagon } ABCDE} = \frac{1}{6}$

Since the whole hexagon is made up of 6 congruent equilateral triangles.

Hints & Solutions

$\therefore$ If area of the hexagon be S, then the area of an equilateral triangle (ΔAPF) is $\frac{S}{6}$.



Now since the area of ΔAOF is half of the area of ΔAPF.

$\therefore$ Area of $\Delta AOF = \frac{1}{2}(\Delta$

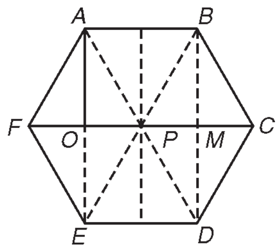
$= \frac{1}{2}\left(\frac{S}{6}\right.$

Hence,
$$\frac{\text{area of } \Delta AOF}{\text{area of hexagon } ABCDEF} = \frac{1}{12}$$

Alternatively : In the given figure there are 12 triangles congruent to ΔAOF *i.e.*, the whole hexagon is the combination of 12 triangles exactly similar to ΔAOF.

Hence, $\frac{\text{area of } \Delta AOF}{\text{area of hexagon}} = \frac{1}{12}$

2. Let F and S denote the faster and slower runner.



Since the ratio of speeds of slower is to faster runner is 1 : 2, hence when S completes one round, F completes 2 round of the circular track.

Thus in 5 minutes (when they meet for the first time) S has completed one round (of 1000 meter) and F has completed two rounds and hence traversed 2000 m. Thus F needs $5 \times 2 = 10$ minutes to traverse $2000 \times 2 = 4000$ m.

3. $-1 \le v \le 1$, $-2 \le u \le -0.5$ and $-2 \le z \le -0.5$

$$w = \frac{vz}{u}$$

For the maximum value of w : Since u is always negative, so $(v.z)$ must be negative ($\because$ – ve ÷ – ve = + ve) in order to get the positive value of w.

Also, to get the maximum positive value of w, numerical value of numerator (*i.e.*, $v.z$) must be greatest and numerical value of denominator (*i.e.*, u) must be least.

Thus $u = -0.5$ and $v = 1$ and $z = -2$

$$\therefore \quad w = \frac{vz}{u} = \frac{1 \times -2}{-0.5} = 4$$

For the minimum value of w : Since u is always negative, so $(v.z)$ must be positive ($\because$ + ve ÷ – ve = – ve) in order to get the negative value of w.

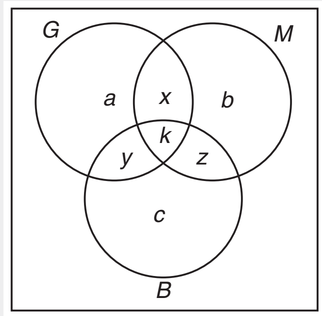
Also, to get the minimum value of w, numerical value of numerator (*i.e.*, $v.z$) must be greatest and numerical value of denominator (*i.e.*, u) must be least.

Thus $u = -0.5$ and $v = -1$ and $z = -2$

$$\therefore \quad w = \frac{vz}{u} = \frac{-1 \times -2}{-0.5} = -4$$

Hence $-4 \le w \le 4$

✍ ***Solutions for question no. 4 and 5 :***



Let α = k
Given

$\Rightarrow$ $x = 8$ *and* $y = 3$ *and* $z = 2$

Also, $(\alpha + \beta + \gamma) = 2(\beta + \gamma) - 1$

$\therefore$ $(\alpha + 19) = 2(19) - 1$

$\Rightarrow$ $\alpha = 18$

$\Rightarrow$ $a + b + c = 18$ ...(1)

Also, $a + c = b + x + k + z = b + 16$

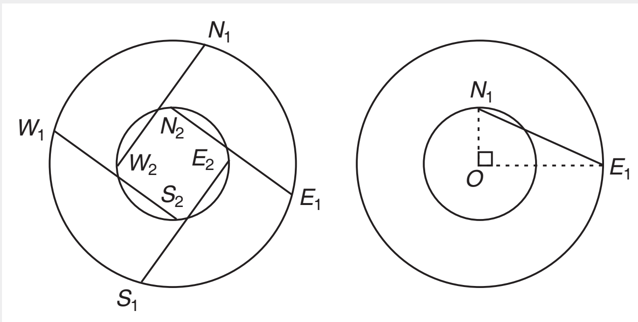
$\Rightarrow \therefore$ $a + b + c = 2b + 16 = 18$ [*from (1)*]

$\Rightarrow$ $b = 1$

4. We have $a + c = b + 16 = 17$.
Further there is no any equation with which $a + c$ can be solved to get the values of a and c individually.
Hence we cannot determine the no. of projects 'a' in which Gyani alone is involved.

5. $\because b = 1$, therefore Medha alone is involved in only one project.

✍ ***Solutions for question no. 6 to 8 :*** *Let the radius of inner circle (IR) be r and the radius of the outer circle (OR) be R and 'O' be the centre of concentric circles.*



Hence length of each chord

$$E_1N_2 = N_1W_2 = W_1S_2 = S_1E_2 = r\sqrt{5}$$

6. $\therefore \frac{\text{Length of all chord roads}}{\text{Length of the outer ring road}} = \frac{4 \times 4\sqrt{5}}{2\pi(2r)} = \frac{\sqrt{5}}{\pi}$

7. Total time $= \frac{90}{60} = \frac{\frac{1}{4}[2\pi(2r)]}{30\pi} + \frac{r\sqrt{5}}{15\sqrt{5}}$

$$\frac{3}{2} = \frac{r}{30} + \frac{r}{15} \Rightarrow r = 15$$

$\therefore$ $R = 2r = 30$ km

8. Total required time $= \frac{r\sqrt{5}}{15\sqrt{5}} + \frac{\frac{1}{2}(2\pi r)}{20\pi}$

$$= \frac{15\sqrt{5}}{15\sqrt{5}} + \frac{\pi \times 15}{20\pi} \quad (\because r = 15 \text{ km})$$

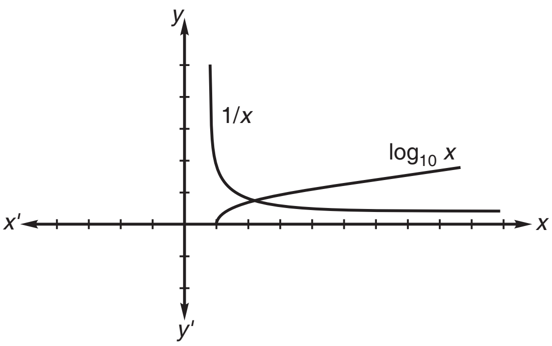
$$= 1 + \frac{3}{4} = \frac{7}{4} \text{ hrs} = 105 \text{ min}$$

9. To get the point(s) of intersection of two curves, we equate the two equation as

$$\log_{10} x = x^{-1} = \frac{1}{x} \Rightarrow 10^{\frac{1}{x}} = x$$

$$\Rightarrow \quad x^x = 10$$

Hence there is only one possible value of x; $(2 < x < 3)$
Therefore the two graphs intersect each other only once.
Alternatively : Plotting the rough sketch of two graphs we get the following diagram :
As we increases the value of x the graph of $\log_{10} x$ diverges



from x-axis and the graph of $\frac{1}{x}$ ($i.e.$, x^{-1}) getting closed to x-axis.
The two graphs are intersecting some where between $x = 2$ and $x = 3$.

10. $T_3 + T_{15} = T_6 + T_{11} + T_{13}$

$\Rightarrow$ $2a + 16d = 3a + 27d$

$\Rightarrow$ $a = -11d$

$\because$ $T_n = 0$

$\Rightarrow$ $a + (n-1)d = 0$

$\Rightarrow$ $-11d + (n-1)d = 0$

$\Rightarrow$ $n = 12$

Hence, T_{12} $i.e.$, 12th term will necessarily be zero.

11. Let the number of correct answers be 'a', number of wrong answers be 'b' and number of questions not attempted be 'c'.

Thus $a + b + c = 50$ ...(1)

$$a - \frac{b}{3} - \frac{c}{6} = 32 \quad ...(2)$$

The second equation can be written as,

$$6a - 2b - c = 192 \quad ...(3)$$

Adding the (1) and (3), we get

$$7a - b = 242$$

$$\Rightarrow \quad a = \frac{(242 + b)}{7}$$

Since 'a' and 'b' both are integers. Thus at $b = 1, 2$, a is not an integer hence at $b = 3$, we obtain 'a' as an integer.
Alternatively : Go through options and try the least valued option first, then higher valued option since it is being asked to find the least possible no. of wrong answers.

	Correct	Wrong	Not attempted	Net Score
No. of questions	35	3	12	32
Marks scored	35	– 1	– 2	

Since the net score of 32 is possible when there are 3 wrong questions, hence the choice (c) is correct.

12. No. of even integers among 100, 102, ...200 = 51
No. of even integers divisible by 7 = 7
(These numbers are : 112, 126, 140, ... 196)
No. of even integers divisible by 9 = 6
(These numbers are : 108, 126, 144 ... 198)
No. of even integers divisible by both 7 and 9 = 1
(The number is 126)
$\therefore$ No. of even integers which are divisible by either 7 or 9 $= (7 + 6 - 1) = 12$
Hence, the required no. of even integers = 51 – 12 = 39

13. Given, $p = x + 2y - 3z$

$q = 2x + 6y - 11z$

$r = x - 2y + 7z$

$\therefore$ $5p = 5x + 10y - 15z$ ...(1)

$2q = 4x + 12y - 22z$ ...(2)

$r = x - 2y + 7z$ ...(3)

Now we can see that option (a) is correct.

$i.e.$, $5p - 2q - r = (5x + 10y - 15z) - (4x + 12y - 22z) - (x - 2y + 7z) = 0$

14. At $x = 2$, $f(x) = 0 + 0.5 + 1.6 = 2.1$

at $x = 2.5$, $f(x) = 0.5 + 0 + 1.1 = 1.6$

at $x = 3.6$, $f(x) = 1.6 + 1.1 + 0 = 2.7$

Thus at $x = 2.5$, $f(x)$ will be minimum.

NOTE If you draw the graphs of $|x - 2|$, $|2.5 - x|$ and $|3.6 - x|$ you will find that at $x < 2$ and $x > 3.6$, values of all the three functions certainly increases.

15. Let the radii of spheres A and B be r_A and r_B.
Since the surface area of B is 300% more than that of A
$\therefore$ the surface area of B is 400% ($i.e.$, 4 times) of area of A.

Hence, $\pi r_A^2 : \pi r_B^2 = 1 : 4$

$\Rightarrow$ $r_A^2 : r_B^2 = 1 : 4$

$\Rightarrow \quad r_A : r_B = 1 : 2$

$\Rightarrow \quad r_A^3 : r_B^3 = 1 : 8$

$\Rightarrow \quad \pi r_A^3 : \pi r_B^3 = 1 : 8$

$\Rightarrow$ Volume of A : Volume of $B = 1 : 8$

$\therefore \quad \dfrac{\text{Volume}(B) - \text{Volume}(A)}{\text{Volume}(B)} \times 100 = \dfrac{8-1}{8} \times 100 = 87.5\%$

Hence volume of sphere A is 87.5% less than the volume of sphere A.

16. Statement (a) can be true, for example it may be no one but the host or some other person be acquainted with all the rest 26 guests.
Statement (c) can be true, for example there is a very new friend of the host who knows only to the host (an odd number of acquaintance).
Statement (d) can also be true that there is no set of three mutual acquantances.
Hence only choice (b) is the most appropriate answer.

17. Please note that we are required to find out the minimum value of the $g(x)$, but $g(x)$ always prefer to give the maximum of the two values ($i.e.,$ $5 - x$, $x + 2$).
Thus there is only one possible condition to get the minimum of $g(x)$ that is when both the values $i.e.,$ $5 - x$ and $x + 2$ are equal.

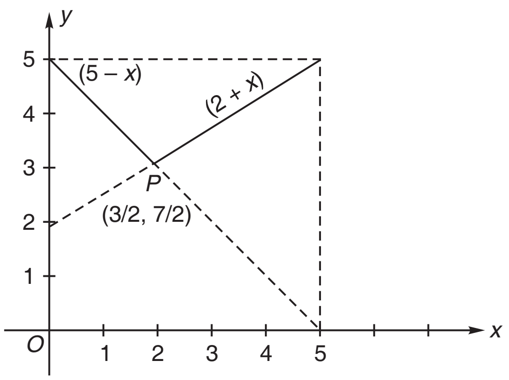
$\therefore \quad 5 - x = x + 2$

$\Rightarrow \quad x = \dfrac{3}{2}$

Thus at $x = \dfrac{3}{2}$, we get minimum value of

$$g(x) = (3.5, 3.5) = 3.5$$

Alternatively : From the graph it is clear that at $P\left(\dfrac{3}{2}, \dfrac{7}{2}\right)$,



$i.e.,$ at $x = \dfrac{3}{2}$, $y = \dfrac{7}{2}$ is minimum.

Please note that as per the given function max $(5 - x, x + 2)$ we will consider always greater values out of the two $(5 - x, x + 2)$ for every x.

18. Since in the base 2 notation, base 3 notation and base 5 notation, last digit ($i.e.,$ unit digit) is 1, hence the required nured number must leave the remainder 1 in each case when divided by either 2, 3 or 5.
Thus there are only 2 choices left to check choice (a) and choice (d)

2 | 91	3 | 91	5 | 91
2 | 45 → 1	3 | 30 → 1	5 | 18 → 1
2 | 22 → 1	3 | 10 → 0	5 | 3 → 3
2 | 11 → 0	3 | 3 → 1	0 → 3
2 | 5 → 1	3 | 1 → 0	
2 | 2 → 1	0 → 1	
2 | 1 → 0		
0 → 1		

$\therefore \quad (91)_{10} \rightarrow (1011011)_2$

$\therefore \quad (91)_{10} \rightarrow (10101)_3$

$\therefore \quad (91)_{10} \rightarrow (331)_5$

Hence in each of the three notations last digit is one and out of 3 cases, there are exactly two cases in which leading digit (*i.e., MSD*) is 1.
Hence choice (d) is correct.

✍ ***Solutions for question no. 19 and 20 :***

Cost of all the three bottles

$= 520 + 364 + 364 = 1248$

$\therefore$ *Share of each person* $= \dfrac{1248}{3} = 416$

Amount spent by $R = 2$ *Euros* $= 92$ *Bahts*

Amount spent by $M = 4$ *Euros* $+ 27$ *Bahts* $= 211$ *Bahts*

	R	***M***
Paid amount	92 *Bahts*	211 *Bahts*
Amount	416 – 92	416 – 211
Payable to S	324 *Bahts*	205 *Bahts*

But 205 Bahts = 5 US Dollars.

19. R owes 324 Bahts to S.

20. M owes 5 US Dollars to S.

21. Go through options.
Clearly choice (c) and (d) are wrong since to process the bags on machine B there is no sufficient time available.
Now consider choice (a)
Total profit $= 75 \times 20 + 80 \times 30 = 3900$
For choice (b)
Total profit $= 100 \times 20 + 60 \times 30 = 3800$
Hence, choice (a) is correct since it gives maximum profit under the given conditions.

22. $p > q$
For the clarification of the concept once again study the chapter : **Percentages.** It is a very fundamental concept of percentage.

e.g., 100 $\underset{-q\%}{\overset{+p\%}{\rightleftarrows}}$ $(100 + p)$ or 100 $\underset{20\%}{\overset{+25\%}{\rightleftarrows}}$ 125

In this case $p > q$ (always)

23. No. of concave corners $= (n - 4) = 25 - 4 = 21$

where n is the number of convex corners.

24. $p + q = (\alpha - 2)$ and $pq = -(\alpha + 1)$

$\therefore \quad (p+q)^2 = p^2 + q^2 + 2pq$

$(\alpha - 2)^2 = p^2 + q^2 + 2[-(\alpha + 1)]$

$\Rightarrow \quad p^2 + q^2 = \alpha^2 - 2\alpha + 6 = (\alpha^2 - 2\alpha + 1) + 5$

$\Rightarrow \quad p^2 + q^2 = (\alpha - 1)^2 + 5$

Hence the minimum value of $p^2 + q^2$ is 5.

25. Sum of first 'n' natural numbers $= \dfrac{n(n+1)}{2}$

$\therefore$ The sum of first 23 natural numbers $= 23 \times 12 = 276$

and the sum of first 24 natural numbers $= \dfrac{24 \times 25}{2} = 300$

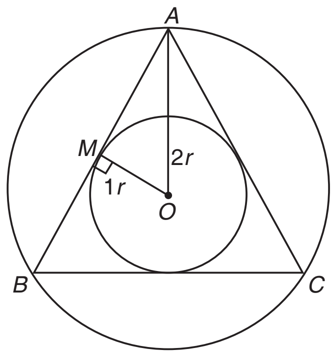
Hence the 288th term of the sequence will be 24th letter of the English alphabet *i. e.*, x.

HINT If we express the sequence in the subsets $S_1, S_2, S_3 \ldots S_n$ then the last element of nth subset is given by $\dfrac{n(n+1)}{2}$ *i.e.*, sum of first 'n' natural numbers.

$S_1 = \{a\}$
$S_2 = \{b, b\}$
$S_3 = \{c, c, c\}$
$S_4 = \{d, d, d, d\}$
$S_5 = \{e, e, e, e, e\}$
$S_6 = \{f, f, f, f, f, f\}$ etc.

i.e., $S_1, S_2, S_3 \ldots S_n$
$\{a\}\ \{b, b\}\{c, c, c\}\ \{d, d, d, d\}\ \{e, e, e, e, e\}\ \{f, f, f, f, f, f\} \ldots$ etc.

26.



Since the area of outer circle is 4 times than the area of inner circle.
Therefore the radius of the outer circle (*i. e.*, circumradius) will be two times the inner circle (*i. e.*, inradius)
Since $AB = AC \therefore \angle ABC = \angle ACB$.

Also $\quad \angle MAO = \sin\theta = \dfrac{r}{2r} = \dfrac{1}{2}$

$\Rightarrow \quad \theta = 30°$

$\therefore \quad \angle BAC = \angle MAC = 2(\angle MAO) = 60°$

Hence, the given triangle is an equilateral triangle.
Alternatively : When the ratio between inradius and circumradius is 1 : 2 then the triangle lies between incircle and circumcircle is an equilateral triangle.

Now, the radius of circumcircle $= R = (2r) = \sqrt{\dfrac{\text{area}}{\pi}} = \sqrt{\dfrac{12}{\pi}}$

$\therefore \quad$ Height of the triangle $= \dfrac{3}{2} \times \sqrt{\dfrac{12}{\pi}}$

$\therefore \quad$ Each side of the triangle $= \dfrac{2}{\sqrt{3}}\left(\dfrac{3}{2} \times \sqrt{\dfrac{12}{\pi}}\right) = \dfrac{6}{\sqrt{\pi}}$

$\therefore \quad$ Area of the triangle $= \dfrac{\sqrt{3}}{4} \times \left(\dfrac{6}{\sqrt{\pi}}\right)^2 = \dfrac{9\sqrt{3}}{\pi}$

27. Given, $a + b + c + d = 4m + 1$

$\Rightarrow a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd) = (4m+1)^2$

Since we know that if $a + b + c + d = k$ (constant) then the product of any two or more of them (*i. e.*, a, b, c, d) will be maximum.
Hence, when $(ab + ac + ad + bc + bd + cd)$ is maximum then $a^2 + b^2 + c^2 + d^2$ is minimum.
Also, in order to "$ab + ac + ad + bc + bd + cd$" be maximum $a = b = c = d$.

Now, $\quad a + b + c + d = 4m + 1$

$\Rightarrow \quad 4a = 4b = 4c = 4d = 4m + 1$

$\Rightarrow \quad a = b = c = d = m + \dfrac{1}{4} = m + 0.25$

$\therefore 2(ab + ac + ad + bc + bd + cd)$
$= 2(6a^2) = 2(6b^2) = 2(6c^2) = 2(6d^2)$
$= 12(m + 0.25)^2$
$= 12m^2 + 6m + 0.75$

Hence, for the minimum value of $a^2 + b^2 + c^2 + d^2$,
$a^2 + b^2 + c^2 + d^2 + 2(ab + ac + ad + bc + bd + cd)_{max} = (4m+1)^2$

$\Rightarrow (a^2 + b^2 + c^2 + d^2) + 12m^2 + 6m + 0.75 = 16m^2 + 8m + 1$

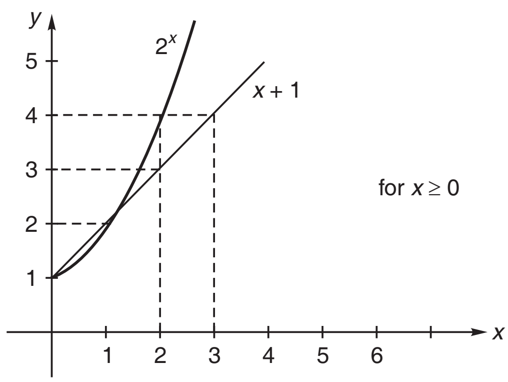
$\Rightarrow \quad a^2 + b^2 + c^2 + d^2 = 4m^2 + 2m + 0.25$

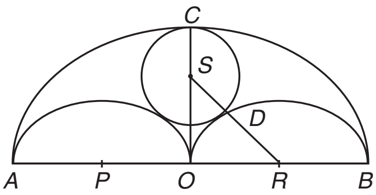
Since a, b, c, d are integers, hence m is also an integer.
Therefore $a^2 + b^2 + c^2 + d^2$ is an integer and hence $4m^2 + 2m + 0.25$ must be integer.
Thus the minimum value of

$a^2 + b^2 + c^2 + d^2 = 4m^2 + 2m + 1.$

Alternatively : Consider some positive value of m, then check the correct option using the logic applied in the above solution.

28. At $x = 0, 1$ the given equation satisfies.
Hence there are only two roots possible.
See the following graph :





$$2^x - x - 1 = 0 \Rightarrow 2^x = x + 1$$

29. Let the radius of largest sphere be $AO = BO = r_1$

$\therefore$ Radii of the semicircles with centres P and $R = AP = PO$

$$DR = OR = RB = \frac{r_1}{2}$$

Let r_2 be the radius of circle with centre S, then $CS = SD = r_2$

Again $\quad OS = CO - CS = r_1 - r_2$

and $\quad SR = SD + DR = r_2 + \frac{r_1}{2}$

$\therefore \quad (SR)^2 = (OS)^2 + (OR)^2$

$\Rightarrow \quad \left(r_2 + \frac{r_1}{2}\right)^2 = (r_1 - r_2)^2 + \left(\frac{r_1}{2}\right)^2$

$\Rightarrow \quad r_1 = 3r_2$

Total area of semicircle with diameter $AB = \frac{1}{2}\pi r_1^2$

Total area of two semicircles with diameters AO and BO

$$= 2 \times \left[\frac{1}{2}\pi\left(\frac{r_1}{2}\right)^2\right]$$

and the area of circle with centre $S = \pi r_2^2 = \pi\left(\frac{r_1}{3}\right)^2$

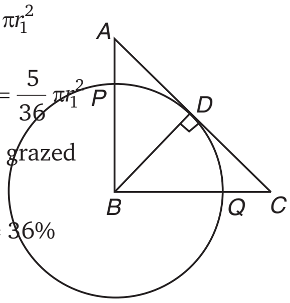
$\therefore$ Total area that can be grazed by horses

$$\frac{\pi r_1^2}{4} + \frac{\pi r_1^2}{9} = \frac{13}{36}\pi r_1^2$$

$\therefore$ Ungrazed area $= \frac{\pi r_1^2}{2} - \frac{13}{36}\pi r_1^2 = \frac{5}{36}\pi r_1^2$

$\therefore$ Percentage area that cannot be grazed

$$= \frac{\frac{13}{36}\pi r_1^2}{\pi r_1^2} \times 100 = 36\%$$



30. $BP = BD = BQ = r$ (radius)

$\therefore \quad BD = \frac{AB \times BC}{AC} = \frac{6 \times 8}{10} = 4.8$

$\therefore \quad AP = AB - BP$

$= 6 - 4.8 = 1.2$ cm

and $\quad CQ = BC - BQ$

$= 8 - 4.8 = 3.2$ cm

$\therefore \quad \frac{AP}{QC} = \frac{1.2}{3.2} = \frac{3}{8}$

31. $\Delta APB \sim \Delta DPC \quad (\because \angle APB = \angle CPD$ and $\angle ABC = \angle PCD)$

$\therefore \quad \frac{AB}{CD} = \frac{BP}{CP} = \frac{3}{1}$

Again $\Delta BCD \sim \Delta BPQ \quad (\because \angle B$ is common and $PQ \parallel CD)$

$\therefore \quad \frac{BP}{CP} = \frac{BQ}{DQ} = \frac{3}{1}$

$\therefore \quad \frac{BD}{BQ} = \frac{4}{3} = \frac{CD}{PQ} \quad (\because BD = BQ + DQ)$

$\therefore \quad CD : PQ = 4 : 3 = 1 : 0.75$

32. Go through options.

Remember, If $\log_a x$, $\log_b y$, $\log_c z$ are in AP then x, y, z are in GP

Hence choice (d) is correct.

$\log_3 2, \log_3(2^3 - 5), \log_3\left(2^3 - \frac{7}{2}\right)$

$\Rightarrow \log_3 2, \log_3 3, \log_3(4.5)$

Since 2, 3, 4.5 are in GP

Threfore the required value of $x = 3$.

33. Let $\quad \angle ACD = \angle BCD = 20° \quad$ (considered arbitrarily)

$\therefore \quad \angle BOC = \angle BCO = 20° \quad (\because BO = OC)$

$\therefore \quad \angle ABO = 40° = \angle BAO \quad (\because AO = BO)$

$\therefore \quad \angle AOB = 100°$

$\therefore \quad \angle AOD = 180° - (\angle AOB + \angle BOC)$

$= 60°$

$\angle AOD = 3 \cdot (\angle ACD)$

$\Rightarrow \quad x = 3y$

$\Rightarrow \quad k = 3$

Alternatively : $\because BO = BC$

$\therefore \quad \angle BOC = \angle BCO = \angle ACD = y$

$\therefore \quad \angle ABO = 2y = \angle OAB \quad (\because AO = BO)$

and ($\angle ABO$ is the external angle)

$\therefore \quad \angle AOB = 180° - (2y + 2y) = 180 - 4y$

$\therefore \quad \angle AOD = 180° - (\angle AOB + \angle BOC)$

$\Rightarrow \quad x = 180° - (180° - 4y + y)$

$\Rightarrow \quad x = 3y$

$\Rightarrow \quad k = 3$

34. $\because x \neq 0 \therefore$ least possible value of $x = 1$ but z can be equal to zero.

$\therefore$ least possible value of $y = 2$; $\quad [\because y > (x, z)]$

y	x	z	$x \times y \times z$	No. of numbers
2	1	0, 1	$1 \times 1 \times 2$	2
3	1, 2	0, 1, 2	$2 \times 1 \times 3$	6
4	1, 2, 3	0, 1, 2, 3	$3 \times 1 \times 4$	12
5	1, 2, 3, 4,	0, 1, 2, 3, 4	$4 \times 1 \times 5$	20
6	1, 2, 3, 4, 5	0, 1, 2, 3, 4, 5	$5 \times 1 \times 6$	30
7	1, 2, 3, 4, 5, 6	0, 1, 2, 3, 4, 5, 6	$6 \times 1 \times 7$	42
8	1, 2, 3, 4, 5, 6, 7	0, 1, 2, 3, 4, 5, 6, 7	$7 \times 1 \times 8$	56
9	1, 2, 3, 4, 5, 6, 7, 8	0, 1, 2, 3, 4, 5, 6, 7, 8	$8 \times 1 \times 9$	72
			Total	240

35. No. of balls in first, second, third, fourth layers … etc. is 1, 3, 6, 10, 15, 21, 28, 36, 45, 55 etc. respectively.

$\therefore$ Total no. of balls in top 1, top 2, top 3, top 4 layers etc. is 1, 4, 10, 20, 35, 56, 84, … etc. respectively.

Thus if 'n' is the total number of balls in top 6, top 16, top 26, top 36, top 46 layers etc, then the unit digit of n is 6.
Also if n is the total number of balls in top 3, top 13, top 23, top 33, top 43 layers etc,
then the unit digit of n is 6.
But as per the given choices only 36 is the suitable value of n.
Hence choice (c) is correct.

Alternatively : No. of balls in first, second, third … nth layer is 1, 3, 6, … $\frac{n(n+1)}{2}$.

$\therefore$ Sum of all the balls in top first, top second, … top n layers is

$$= 1 + 3 + 6 + \ldots \frac{n(n+1)}{2}$$

$$= \Sigma\left[\frac{n(n+1)}{2}\right] = \frac{1}{2}[\Sigma\,(n^2 + n)]$$

$$= \frac{1}{2}[\Sigma\, n^2 + \Sigma\, n]$$

$$= \frac{1}{2}\left[\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2}\right]$$

$$= \frac{1}{2}\cdot\frac{n(n+1)}{2}\left[\frac{2n+1}{3} + 1\right]$$

$$= \frac{n(n+1)(n+2)}{6}$$

$$\therefore \quad \frac{n(n+1)(n+2)}{6} = 8436$$

$$\Rightarrow \quad n = 36$$

36. Let $n = 2$ such that $2 \times \frac{1}{2} = 1$

$$\therefore \quad 2 + \frac{1}{2} = \frac{5}{2}$$

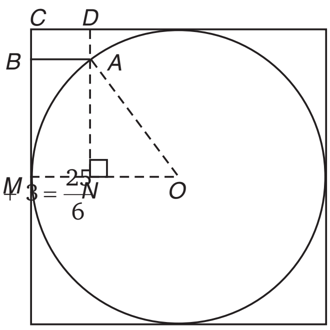
hence choices (a) and (d) are eliminated.

Again if $n = 3$, such that $\frac{1}{2} \times \frac{2}{3} \times 3 = 1$

$$\therefore \quad \frac{1}{2} + \frac{2}{3} + 3 = \frac{25}{6}$$

Thus choice (b) is also eliminated.

Hence the correct choice is (c).



37. Draw the perpendiculars OM and AN as shown in figure and join the points A and O, where O is the centre of circle.

$$(OA)^2 = (AN)^2 + (ON)^2$$

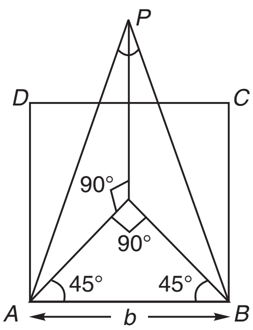
$$\Rightarrow (OA)^2 = (DN - DA)^2 + (MO - MN)^2$$

$$\Rightarrow r^2 = (r - 10)^2 + (r - 20)^2$$

$(CM = DN = OM = OA = r)$

$$\Rightarrow \quad r = 50$$

Alternatively : Go through options.



38. $AB = b$

$$\therefore \quad OA = OB = \frac{b}{\sqrt{2}}$$

($\because \Delta\, AOB$ is a right angled)

Also $\angle APB = 60°$ and $\Delta\, APB$ is an equilateral triangle. ($\because AP = BP$)

$$\therefore \quad AP = AB = BP = b$$

Now, $OP = \sqrt{AP^2 - OA^2}$

($\because \Delta\, APO$ is right angled)

$$h = \sqrt{b^2 - \left(\frac{b}{\sqrt{2}}\right)^2}$$

$$h = \frac{b}{\sqrt{2}}$$

$$\Rightarrow \quad b^2 = 2h^2$$

39. b can be either of 2, 4, 6, 8, … etc.
So we cannot determine the relation by the first statement itself.
Now, since $b > 16$; $b \in I \therefore b = 17, 18, 19, \ldots$ etc.
Thus we can determine the unique relation by the second statement itself.
As. $a^{44} = 2^{44}$ and $b^{11} = 17^{11}$ or 18^{11} or 19^{11} … etc.

$\Rightarrow \quad a^{44} = 16^{44}$ and $b^{11} = 17^{11}$ (least possible value)

hence $16^{44} < 17^{11}$ or 18^{11} or 19^{11} … etc.

Thus the question can be answered by second statement alone but not by the first statement. Hence choice (a) is correct.

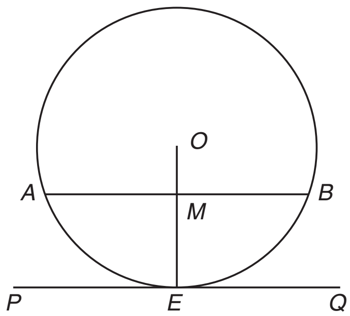
40. Let $\alpha = -\frac{1}{2}$ and $\beta = \frac{1}{2}$ [from statement (A)]

$\therefore \quad \alpha\beta = \frac{c}{a}$ and $\alpha + \beta = -\frac{b}{a}$

$-\frac{1}{4} = \frac{c}{4}$ and $-\frac{1}{2} + \frac{1}{2} = -\frac{b}{a}$

$\Rightarrow \quad c = -1 \quad \Rightarrow \quad b = 0$

Hence, statement (A) alone is sufficient to determine the unique values of b and c.



Now, $b = c$ $\left[\because \frac{c}{b} = 1;\text{ statement (B)}\right]$

$$\therefore \quad 4x^2 + bx + b = 0$$

$$\Rightarrow \quad 4\left(-\frac{1}{2}\right)^2 + b\left(-\frac{1}{2}\right) + b = 0 \qquad \left(\because \alpha = -\frac{1}{2}\right)$$

$$\Rightarrow \quad b = -2, \text{ also } c = -2 \qquad (\because b = c)$$

Hence, statement (B) alone is sufficient to determine the unique values of b and c.
Hence, choice (b) is correct.

41. Statement (A) is not sufficient itself.
Now, consider statement (B)

Let $OE = r = OA = OB$

$\therefore \quad OM = (r - 5) \qquad (\because ME = 5 \text{ cm})$

and $AM = BM = \dfrac{AB}{2} = 2.5 \text{ cm}$

$\therefore \quad OA^2 = AM^2 + OM^2$

$r^2 = (2.5)^2 + (r - 5)^2$

$\Rightarrow \quad r = 3.125 \text{ cm} = OE$

But OE cannot be less than OM, hence data is inconsistence.
But the information given by statement (B) is sufficient to answer.
Hence, choice (a) is correct.

42. Consider statement (A)

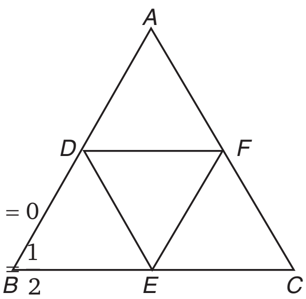
$$\text{Sum of the first series} = \frac{\frac{1}{a^2}}{1 - \left(\frac{1}{a^2}\right)} = \frac{1}{a^2 - 1}$$

$$\text{Sum of the second series} = \frac{\frac{1}{a}}{1 - \left(\frac{1}{a^2}\right)} = \frac{a}{a^2 - 1}$$

From the first statement (*i. e.*, $-3 \le a \le 3$) the relation cannot be determined uniquely since it is different for the +ve and –ve values.
Now, consider statement (B)

$4a^2 - 4a + 1 = 0$

$\Rightarrow \quad a = \dfrac{1}{2}$



Thus for $a = \dfrac{1}{2}$ (a unique value) the sum of the second series will always be greater than that of the first.
Hence, choice (a) is correct.

43. Since D, E, F are the mid points of AB, BC and AC respectively.
Therefore $AB = 2EF$,
$BC = 2DF$, $AC = 2DE$.
Hence the area of

$\Delta DEF = \dfrac{1}{4}(\Delta ABC)$

Consider statement (A)

$AD = 1$ cm $\therefore AB = 2$ cm and $EF = 1$ cm

$DF = 1$ cm $\therefore BC = 2$ cm

Perimeter of $\Delta DEF = 3$ cm

$\therefore \quad DE = 1$ cm, hence $AC = 2$ cm

$\because \quad DE = EF = DF$

$\therefore DEF$ is an equilateral triangle of side 1 cm each.
Thus area can be calculated.
Consider statement (B)
Perimeter of $\Delta ABC = 6$ cm, $\therefore$ Perimeter of $\Delta DEF = 3$ cm

$AB = 2 \text{ cm} \Rightarrow EF = 1 \text{ cm}$

$AC = 2 \text{ cm} \Rightarrow DE = 1 \text{ cm}$

$\therefore \quad DF = 1 \text{ cm} \qquad (\because DE + EF + DF = 3 \text{ cm})$

Hence area can be calculated.
Therefore, choice (b) is correct.

44. When n is a prime number than $(n-1)(n-2)\ldots 3\cdot 2\cdot 1$ is not divisible by n.
Now, since there are 7 prime numbers (13, 17, 19, 23, 29, 31, 37) from 12 to 40 therefore $n = 7$.
e.g., $n = 13$, then $(n-1)(n-2)\ldots 3\cdot 2\cdot 1 = 12!$
Thus $12!$ is not divisible by 13.
Hence, choice (b) is correct.

45. $$\frac{x^2(y+z) + y^2(x+z) + z^2(x+y)}{xyz}$$

$$\Rightarrow \frac{x^2(y+z)}{xyz} + \frac{y^2(x+z)}{xyz} + \frac{z^2(x+y)}{xyz}$$

$$\Rightarrow \left(\frac{x}{y} + \frac{x}{z}\right) + \left(\frac{y}{x} + \frac{y}{z}\right) + \left(\frac{z}{x} + \frac{z}{y}\right)$$

$$\Rightarrow \left(\frac{x}{y} + \frac{y}{x}\right) + \left(\frac{y}{z} + \frac{z}{y}\right) + \left(\frac{z}{x} + \frac{x}{z}\right)$$

Now, since $\dfrac{a}{b} + \dfrac{b}{a} > 2$; if a and b are distinct numbers.

$$\therefore \left(\frac{x}{y} + \frac{y}{x}\right) + \left(\frac{y}{z} + \frac{z}{y}\right) + \left(\frac{z}{x} + \frac{x}{z}\right) > 2 + 2 + 2 > 6$$

Hence, choice (c) is correct.

46. No. of students who answered 1 or more questions wrongly $= 2^{n-1}$

No. of students who answered 2 or more questions wrongly $= 2^{n-2}$

No. of students who answered 3 or more questions wrongly $= 2^{n-3}$

… … …
… … …

No. of students who answered n questions wrongly $= 2^{n-n} = 1$

Hence total no. of wrong answers

$$= 2^{n-1} + 2^{n-2} + 2^{n-3} + \ldots 2^2 + 2^1 + 1$$

$$4095 = \frac{1\cdot(2^n - 1)}{2 - 1} = 2^n - 1$$

$\Rightarrow \quad 2^n = 4096 = 2^{12}$

$\Rightarrow \quad n = 12$

47. For the intersection of two curves we equate them and get the solution as follows.

$$x^3 + x^2 + 5 = x^2 + x + 5$$

$\Rightarrow \quad x^3 - x = 0$

$\Rightarrow \quad x(x^2 - 1) = 0$

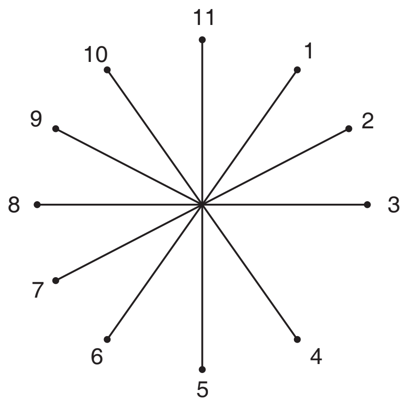
$\Rightarrow \quad x(x+1)(x-1) = 0$

$\Rightarrow \qquad x = 0, -1, 1$

Thus there are three solutions each lying in the given range (*i.e.*, $-2 \le x \le 2$).
Hence, the two curves intersect thrice.

NOTE For more clarification of the concept draw the two graphs on the same plane and you will find that the two graphs intersect each other at $x = -1$, $x = 0$ and $x = 1$ *i.e.*, the corresponding values of y for each x of the two curves are same.

48.
$$3 + 467 = 470$$
$$11 + 459 = 470$$



$$19 + 451 = 470$$
$$27 + 443 = 470$$
$$\cdots \quad \cdots \quad \cdots$$
$$\cdots \quad \cdots \quad \cdots$$

Thus there are 29 pairs which gives the sum of 470 and a single number (which is the 30th number) 243 which never gives the sum of 470 by combining with any other number of the set T.

Therefore we can take 29 numbers (one from each pair) alongwith 243.
Thus the maximum numbers in the subset S can be $29 + 1 = 30$.

49. The least no. of edges will be when 11 points will be connected to a single point through the edges. Hence this combination will give us least possible 11 edges.
The maximum no. of edges will be when all the 12 points will be non collinear and connected with each other.
Hence the maximum no. of edges $= {}^{12}C_2 = 66$

50. No. of ways of filling 1 green ball = 6 (1, 2, 3, 4, 5, 6)

No. of ways of filling 2 green balls = 5
[(1, 2); (2, 3); (3, 4); (4, 5); (5, 6)]

No. of ways of filling 3 green balls = 4
[(1, 2, 3); (2, 3, 4); (3, 4, 5); (4, 5, 6)]

No. of ways of filling 4 green balls = 3
[(1, 2, 3, 4); (2, 3, 4, 5); (3, 4, 5, 6)]

No. of ways of filling 5 green balls = 2
[(1, 2, 3, 4, 5); (2, 3, 4, 5, 6)]

No. of ways of filling 6 green balls = 1 [(1, 2, 3, 4, 5, 6)]

Hence, the required no. of ways = 6 + 5 + 4 + 3 + 2 + 1 = 21