CAT SOLVED PAPER

No. of Questions : 35 *Time :* 40 *min*

✍ ***Instructions :*** *This paper is divided into two sections, section A and section B.*

[Section A : Number of question = 20]

NOTE Question number 1 to 20 carry one mark each and wrong answer carry $\frac{1}{3}$ negative mark.

✍ ***Directions for question number* 1 *to* 14 :** *Answer the questions independently of each other.*

1. Two boats, travelling at 5 and 10 kms per hour, head directly towards each other. They begin at a distance of 20 kms from each other. How far apart are they (in kms) one minute before they collide ?

(a) $\frac{1}{12}$ (b) $\frac{1}{6}$
(c) $\frac{1}{4}$ (d) $\frac{1}{3}$

2. A rectangular sheet of paper, when halved by folding it at the mid point of its longer side, results in a rectangle, whose longer and shorter sides are in the same proportion as the longer and shorter sides of the original rectangle. If the shorter side of the original rectangle is 2, what is the area of the smaller rectangle ?

(a) $4\sqrt{2}$ (b) $2\sqrt{2}$
(c) $\sqrt{2}$ (d) none of these

3. If the sum of the first 11 terms of an arithmetic progression equals that of the first 19 terms, then what is the sum of the first 30 terms ?

(a) 0 (b) – 1
(c) 1 (d) not unique

4. If a man cycles at 10 km/hr, then he arrives at a certain place at 1 p.m. If he cycles at 15 km/hr, he will arrive at the same place at 11 a.m. At what speed must he cycle to get there at noon ?

(a) 11 km/hr (b) 12 km/hr
(c) 13 km/hr (d) 14 km/hr

5. On January 1, 2004 two new societies S_1 and S_2 are formed, each with n members. On the first day of each subsequent month, S_1 adds b members while S_2 multiplies its current number of members by a constant factor r. Both the societies have the same number of members on July 2, 2004. If $b = 10.5n$, what is the value of r ?

(a) 2.0 (b) 1.9
(c) 1.8 (d) 1.7

6. If $f(x) = x^3 - 4x + p$ and $f(0)$ and $f(1)$ are of opposite signs, then which of the following is necessarily true ?

(a) $-1 < p < 2$ (b) $0 < p < 3$
(c) $-2 < p < 1$ (d) $-3 < p < 0$

7. Suppose n is an integer such that the sum of the digits of n is 2, and $10^{10} < n < 10^{11}$. The number of different values for n is :

(a) 11 (b) 10
(c) 9 (d) 8

8. A milkman mixes 20 litres of water with 80 litres of milk. After selling one-fourth of this mixture, he adds water to replenish the quantity that he has sold. What is the current proportion of water to milk ?

(a) 2 : 3 (b) 1 : 2
(c) 1 : 3 (d) 3 : 4

9. $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r$ then r cannot take any value except :

(a) $\frac{1}{2}$ (b) -1
(c) $\frac{1}{2}$ or -1 (d) $-\frac{1}{2}$ or -1

10. Let $y = \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{3 + \dots}}}}$

What is the value of y ?

(a) $\frac{\sqrt{13} + 3}{2}$ (b) $\frac{\sqrt{13} - 3}{2}$
(c) $\frac{\sqrt{15} + 3}{2}$ (d) $\frac{\sqrt{15} - 3}{2}$

11. Karan and Arjun run a 100 metre race, where Karan beats Arjun by 10 metres. To do a favour to Arjun, Karan starts 10 metres behind the starting line in a second 100 metre race. They both run at their earlier speeds. Which of the following is true in connection with the second race ?

(a) Karan and Arjun reach the finishing line simultaneously
(b) Arjun beats Karan by 1 metre
(c) Arjun beats Karan by 11 metres
(d) Karan beats Arjun by 1 metre

12. N persons stand on the circumference of a circle at a distinct points. Each possible pair of persons, not standing next to each other, sings a two minute song one pair after the other. If the total time taken for singing is 28 minutes, what is N ?

(a) 5 (b) 7
(c) 9 (d) none of these

13. In Nuts and Bolts factory, one machine produces only nuts at the rate of 100 nuts per minute and needs to be cleaned for 5 minutes after production of every 1000 nuts. Another machine produces only bolts at the rate of 75 bolts per minute and needs to be cleaned for 10 minutes after production of every 1500 bolts. If both the machines start production at the same rate, what is the minimum duration required for producing 9000 pairs of nuts and bolts ?

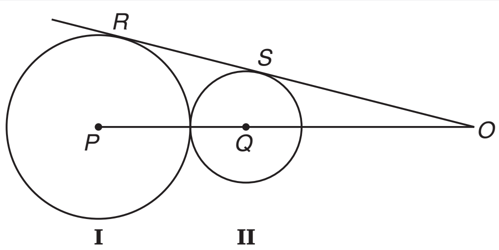
(a) 130 minutes (b) 135 minutes
(c) 170 minutes (d) 180 minutes

14. A father and his son are waiting at a bus stop in the evening. There is a lamp post behind them. The lamp post, the father and his son stand on the same straight line. The father observes that the shadows of his head and his son's head are incident at the same point on the ground. If the heights of the lamp post, the father and his son are 6 metres, 1.8 metres and 0.9 metres respectively and the father is standing 2.1 metres away from the post, then how far (in metres) is the son standing from his father ?

(a) 0.9 (b) 0.75
(c) 0.6 (d) 0.45

✍ ***Directions for question number 15 to 17 :*** *Answer the questions on the basis of the information given below.*
In the adjoining figure I and II are circles with centres P and Q respectively. The two circles touch each other and have a common tangent that touches them at point R and S respectively. This common tangent meets the line joining P and Q at O. The diameters of I and II are in the ratio 4 : 3. It is also known that the length of PO is 28 cm.

15. What is the ratio of the length of PQ to that of QO ?
(a) 1 : 4 (b) 1 : 3
(c) 3 : 8 (d) 3 : 4



16. What is the radius of the circle II ?
(a) 2 cm (b) 3 cm
(c) 4 cm (d) 5 cm

17. The length of SO is :
(a) $8\sqrt{3}$ cm (b) $10\sqrt{3}$ cm
(c) $12\sqrt{3}$ cm (d) $14\sqrt{3}$ cm

✍ ***Directions for question number 18 to 20 :*** *Answer the questions independently of each other.*

18. Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then at $x = 0$, $f(x)$ is :
(a) maximized whenever $a > 0$, $b > 0$
(b) maximized whenever $a > 0$, $b < 0$
(c) minimized whenever $a > 0$, $b > 0$
(d) minimized whenever $a > 0$, $b < 0$

19. Each family in a locality has at most two adults, and no family has fewer than 3 children. Considering all the families together, there are more adults than boys, more boys than girls, and more girls than families. Then the minimum possible number of families in the locality is :
(a) 4 (b) 5
(c) 2 (d) 3

20. The total number of integer pairs (x, y) satisfying the equation $x + y = xy$ is :
(a) 0 (b) 1
(c) 2 (d) none of these

[Section B : Number of questions = 15]

NOTE Question number 21 to 35 carry two marks each and wrong answer carry $\frac{2}{3}$ negative mark.

✍ ***Directions for question number 21 to 24 :*** *Answer the questions independently of each other.*

21. Let C be a circle with centre P_0 and AB be a diameter of C. Suppose P_1 is the mid point of the line segment P_0B, P_2 is the mid point of the line P_1B and so on. Let $C_1, C_2, C_3, \ldots$ be circles with diameters $P_0P_1, P_1P_2, P_2P_3 \ldots$ respectively. Suppose the circles $C_1, C_2, C_3 \ldots$ are all shaded. The ratio of the area of the unshaded portion to C to that of the original circle C is :
(a) 8 : 9 (b) 9 : 10
(c) 10 : 11 (d) 11 : 12

22. Consider the sequence of numbers $a_1, a_2, a_3, \ldots$ to infinity where $a_1 = 81.33$ and $a_2 = -19$ and $a_j = a_{j-1} - a_{j-2}$ for $j \geq 3$. What is the sum of the first 6002 terms of this sequence?
(a) – 100.33 (b) – 30.00
(c) 62.33 (d) 119.33

23. A sprinter starts running on a circular path of radius r metres. Her average speed (in metres/min.) is πr during the first 30 seconds, $\frac{\pi r}{2}$ during next one minute, $\frac{\pi r}{4}$ during next 2 minutes, $\frac{\pi r}{8}$ during next 4 minutes, and so on. What is the ratio of the time taken for the nth round to that for the previous round ?
(a) 4 (b) 8
(c) 16 (d) 32

24. Let $u = (\log_2 x)^2 - 6\log_2 x + 12$ where x is a real number. Then the equation $x^u = 256$, has :
(a) no solution for x
(b) exactly one solution for x
(c) exactly two distinct solutions for x
(d) excatly three distinct solutions for x

✍ ***Directions for question number 25 and 26 :*** *Answer the questions on the basis of the information given below.*

$$f_1(x) = \begin{cases} x & 0 \leq x \leq 1 \\ 1 & x \geq 1 \\ 0 & \text{otherwise} \end{cases}$$

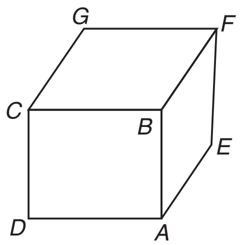
$$f_2(x) = f_1(-x) \quad \textit{for all } x$$
$$f_3(x) = -f_2(x) \quad \textit{for all } x$$
$$f_4(x) = f_3(-x) \quad \textit{for all } x$$

25. How many of the following products are necessarily zero for every x; $f_1(x)f_2(x)$, $f_2(x)f_3(x)$, $f_2(x)f_4(x)$?
(a) 0 (b) 1
(c) 2 (d) 3

26. Which of the following is necessarily true ?
(a) $f_4(x) = f_1(x)$ for all x
(b) $f_1(x) = -f_3(-x) = 0$ for all x
(c) $f_2(-x) = f_4(x)$ for all x
(d) $f_1(x) + f_3(x) = 0$ for all x

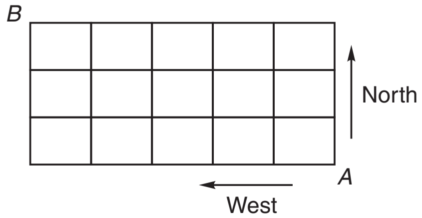
✍ ***Directions for question number 27 to 31 :*** *Answer the questions independently of each other.*

27. If the lengths of diagonals DF, AG and CE of the cube shown in the following figure are equal to the three sides of a triangle, then the radius of the circle circumscribing that triangle will be :



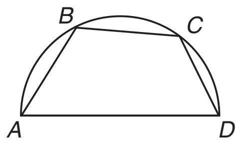
(a) equal to the side of the cube
(b) $\sqrt{3}$ times the side of the cube
(c) $\frac{1}{\sqrt{3}}$ times the side of the cube
(d) impossible to find from the given information

28. In the adjoining figure, the lines represent one-way roads allowing travel only northwards or only westwards :



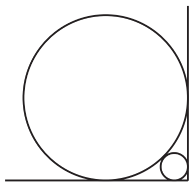
(a) 15 (b) 56
(c) 120 (d) 336

29. On a semicircle with diameter AD, chord BC is parallel to the diameter. Further, each of the chords AB and CD has length 2, while AD has length 8. What is the length of BC ?



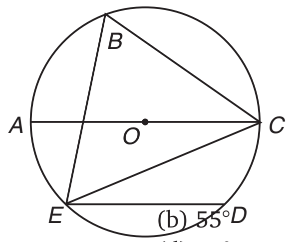
(a) 7.5 (b) 7
(c) 7.75 (d) none of these

30. A circle with radius 2 is placed against a right angle. Another smaller circle is also placed as shown in the adjoining figure. What is the radius of the smallest circle ?



(a) $3 - 2\sqrt{2}$ (b) $4 - 2\sqrt{2}$
(c) $7 - 4\sqrt{2}$ (d) $6 - 4\sqrt{2}$

31. In the adjoining figure, chord ED is parallel to the diameter AC of the circle. If $\angle CBE = 65°$, then what is the value of $\angle DEC$?



(a) 35° (b) 55°
(c) 45° (d) 25°

✍ ***Directions for questions number 32 and 33 :*** *Answer the questions on the basis of the information given below.*
In an examination, there are 100 questions divided into three groups A, B and C such that each group contains at least one question. Each question in group A carries 1 mark, each question in group B carries 2 marks and each question in group C carries 3 marks. It is known that the questions in group A together carry atleast 60% of the total marks.

32. If group B contains 23 questions, then how many questions are there in group C ?
(a) 1 (b) 2
(c) 3 (d) can't be determined

33. If group C contains 8 questions and group B carries atleast 20% of the total marks, which of the following best describes the number of questions in group B ?
(a) 11 or 12 (b) 12 or 13
(c) 13 or 14 (d) 14 or 15

✍ ***Directions for question number 34 and 35 :*** *Answer the questions independently of each other.*

34. The remainder, when $(15^{23} + 23^{23})$ is divided by 19, is :
(a) 4 (b) 15
(c) 0 (d) 18

35. A new flag has to be designed with six vertical stripes using some or all of the colours, green, blue and red. Then the number of ways in which this can be done such that no two adjacent stripes have the same colour is :
(a) 12×81 (b) 16×192
(c) 20×125 (d) 24×216

Answers

1 (c)	**2.** (b)	**3.** (a)	**4.** (b)	**5.** (a)	**6.** (b)	**7.** (a)	**8.** (a)	**9.** (c)	**10.** (d)
11. (d)	**12.** (b)	**13.** (a)	**14.** (d)	**15.** (b)	**16.** (b)	**17.** (c)	**18.** (d)	**19.** (d)	**20.** (c)
21. (d)	**22.** (c)	**23.** (c)	**24.** (b)	**25.** (c)	**26.** (d)	**27.** (a)	**28.** (b)	**29.** (b)	**30.** (d)
31. (d)	**32.** (a)	**33.** (c)	**34.** (c)	**35.** (a)					

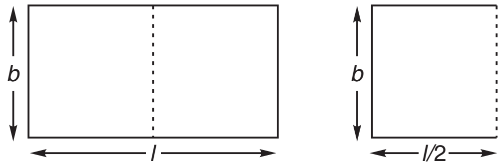
Hints & Solutions

1. Since, two boats are moving in opposite directions towards each other. Therefore their relative speed = 15 km/hr

$$= \frac{1}{4} \text{ km/min.}$$

Hence, before the collision, they must travel $\frac{1}{4}$ km in last one minute. Thus choice (c) is correct.

2. Let the length and breadth of the origional rectangle be l and b respectively.



Then b and $\frac{l}{2}$ are the length and breadth of the smaller rectangle, respectively.

Therefore, we have $\frac{l}{b} = \frac{b}{\frac{l}{2}} \Rightarrow l^2 = 2b^2$

But, $b = 2$, therefore $l = 2\sqrt{2}$

Hence, the area of the smaller rectangle $= b \times \left(\frac{l}{2}\right)$

$$= 2 \times \sqrt{2} = 2\sqrt{2}$$

3. Let a, d be the first term and common difference of the AP respectively.

Given, $S_{11} = S_{19}$

$\Rightarrow \frac{11}{2}[2a + 10d] = \frac{19}{2}[2a + 18d]$

$\Rightarrow 2a + 29d = 0$

$\therefore S_{30} = \frac{30}{2}[2a + 29d] = 15 \times 0 = 0$

4. Since, here distance (*i.e.*, product of speed and time) is constant so, we can use the product constancy concept.

Speed	Time
$\frac{1}{2}\uparrow$	$\frac{1}{3}\downarrow$

Let the initial time be t hours.

Therefore, $\frac{t}{3} = 2 \Rightarrow t = 6$ hours

Alternatively : $10 \times t = 15 \times (t - 2) =$ distance

$\Rightarrow t = 6$ hours

Now, the distance travelled = $10 \times 6 = 60$ or $15 \times 4 = 60$ km

But the new time is 5 hours, hence the required speed $= \frac{60}{5} = 12$ km/hr

5. For the society S_1, the number of members in the given months are as follows :

$$n, n + b, n + 2b, n + 3b, n + 4b, n + 5b, n + 6b$$

and for the society S_2, the number of members in the given months are as follws :

$$n, nr, nr^2, nr^3, nr^4, nr^5, nr^6$$

Given, $n + 6b = nr^6$

$\Rightarrow n + 6 \times 10.5n = nr^6$

$\Rightarrow 64 = r^6$

$\Rightarrow r = 2$

Hence choice (a) is correct.

6. $f(x) = x^3 - 4x + p$

$f(0) = p, \quad f(1) = p - 3$

Given $f(0)$ and $f(1)$ are of opposite signs, therefore one of them will be negative and another will be positive, thus the product of them will be negative.

i.e., $f(0) \times f(1) < 0$

$\Rightarrow p \times (p - 3) < 0$

If $p < 0$, then $p - 3$ is also less than 0.

$\therefore p(p - 3) > 0$ *i.e.*, p can not be negative.

$\therefore$ Choices (a), (c) and (d) are eliminated.

Hence, choice (b) is correct *i.e.*, $0 < p < 3$.

Alternatively : Go through options

Here, $f(0) = 0$ and $f(1) = p - 3$

Choice (a) : Let $p = -0.5$ $(\because -1 < p < 2)$

$\therefore p - 3 = -3.5$,

which shows that p and $p - 3$ both are negative (*i.e.*, same signs). Hence, wrong choice.

Choice (c) : Let $p = -1$ $(\because -2 < p < 1)$

$\therefore p - 3 = -4$, hence wrong.

Choice (d) : Let $p = -2$

$\therefore p - 3 = -5$, hence wrong

Thus, it is clear that choice (b) is correct.

7. The sum of digits can be 2 in the following two ways :

(i) 1 + 1 + 0 + 0 + 0 + ... = 2

(ii) 2 + 0 + 0 + 0 + ... = 2

$\therefore$ The possible numbers lying between

10^{10} (*i.e.*, 10,00,00,00,000)

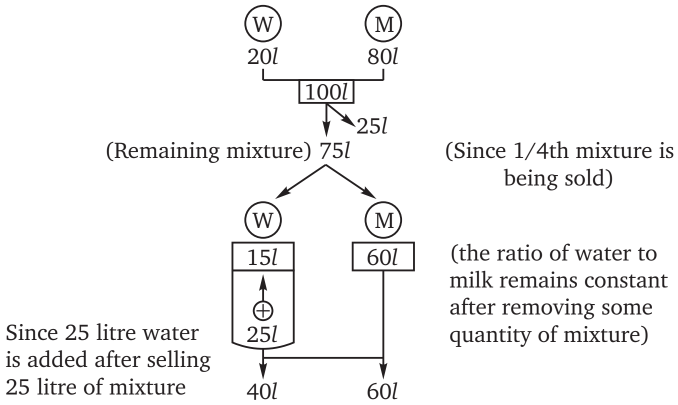
and 10^{11} (*i.e.*, 1,00,00,00,00,000) are

$$\left.\begin{matrix} 10000000001 \\ 10000000010 \\ 10000000100 \\ 10000001000 \\ 10000010000 \\ \ldots \ldots \ldots \ldots \\ \ldots \ldots \ldots \ldots \\ 11000000000 \end{matrix}\right\} 10$$

and $20000000000\}1$

Thus, there are total 10 + 1 = 11 numbers.

8.



Thus, after the replacement there are 40 litre water and 60 litre milk in the mixture. Hence, the required ratio of water to milk is 2 : 3.

Alternatively : Since $\frac{1}{4}$th mixture is taken out, therefore remaining amount of milk in the mixture is $80 - \frac{80}{4} = 60$ litre.

Thus, to complete (*i.e.*, replenish) the initial amount of mixture (*i.e.*, 100 litre) milkman must have 40 litre water (40 = 100 – 60) in the mixture. Hence, the required ratio = 40 : 60 ⇒ 2 : 3.

9. Given : $$\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r$$

$$\Rightarrow \quad \frac{a+b+c}{b+c+c+a+a+b} = r$$

(See the property number 11 in the chapter–Ratio and Proportion)

$$\Rightarrow \quad \frac{a+b+c}{2(a+b+c)} = r$$

$$\Rightarrow \quad \frac{1}{2} = r \qquad (\text{if } a+b+c \neq 0)$$

Now, if $a + b + c = 0$, then

$$\frac{a}{b+c} = -1 \qquad (\because\ a+b+c=0 \Rightarrow b+c=-a)$$

Similarly, $$\frac{b}{c+a} = -1 \qquad (\because\ c+a=-b)$$

and $$\frac{c}{a+b} = -1 \qquad (\because\ a+b=-c)$$

$$\therefore \quad \frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{a+b} = r = -1$$

Hence, choice (c) is correct.

Alternatively : Go through options

Choice (a) : Let $r = \frac{1}{2}$, then we can assume

$$a = b = c = 1$$

then $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{b+a} = r = \frac{1}{2}$ is true

Choice (b) : Let $r = -1$, then we can assume

$$a = -1, \quad b = 2 \quad \text{and} \quad c = -1$$

then $\frac{a}{b+c} = \frac{b}{c+a} = \frac{c}{b+a} = r = -1$, is true

Choice (d) : Let $r = -\frac{1}{2}$, then we can assume

$$a = -1, \quad b = 1 \quad \text{and} \quad c = 1$$

$$\therefore \quad \frac{a}{b+c} \neq \frac{b}{c+a} \neq \frac{c}{b+a}$$

Hence, choice (c) is most appropriate.

10. Since, $$y = \cfrac{1}{2 + \cfrac{1}{3 + \cfrac{1}{2 + \cfrac{1}{3 + \dots}}}}$$

$$\therefore \quad y = \cfrac{1}{2 + \cfrac{1}{3 + y}}$$

$$\Rightarrow \quad y = \frac{3+y}{6+2y+1}$$

$$\Rightarrow \quad 2y^2 + 6y - 3 = 0$$

$$\therefore \quad y = \frac{-6 \pm \sqrt{36+24}}{4}$$

$$\Rightarrow \quad y = \frac{-3 \pm \sqrt{15}}{2}$$

Since, y is a positive value, hence

$$y = \frac{-3+\sqrt{15}}{2} \quad \text{or} \quad y = \frac{\sqrt{15}-3}{2}$$

Alternatively : Since, $y = \frac{1}{2+k}$, k is some positive value

Then y must be a positive value less than $\frac{1}{2}$

Hence, choices (a) and (c) are obviously inadmissible.

Also, the value of k must be less than $\frac{1}{3}$.

Hence, the y must be greater than 0.375 $\left(\cfrac{1}{2+\cfrac{1}{3}} = 0.375\right)$

Therefore, choice (b) is also ruled out.

Hence, choice (d) is correct

11. As per the data given about the first race, when Karan runs 100 m, then Arjun runs only 90 m. Thus, the ratio of speeds of Karan and Arjun is 100 : 90 ⇒ 10 : 9

Thus, in the second race when Karan runs 110 m (= 10 + 100) then Arjun will run only 99 m $\left(= \frac{9}{10} \times 110\right)$

i.e., less than100 m.

Hence, Karan beats Arjun by 1 m.

Alternatively : Since time is constant, therefore distance is directly proportional to speed.

Hence, $$\frac{\text{Karan's Speed}}{\text{Arjun's Speed}} = \frac{100}{90} = \frac{110}{x}$$

$$\Rightarrow \quad x = 99$$

Thus, when Karan runs 110 m, Arjun runs only 99 m which is 1 m less than the length of race course.

Hence, Karan beats Arjun by 1 meter.

12. When N persons stand on the circumference of a circle they can form an N sided polygon when they are joined directly. Also, we know that when the adjacent points are joined directly they become the vertices of the polygon. When all these points are joined with each other then there must be ${}^NC_2$ lines (all the points must be non-collinear). But if we do not consider the joining of a vertex with its two adjacent vertices then we have only diagonals of the polygon which is equal to

$${}^NC_2 - N = \frac{N(N-3)}{2}$$

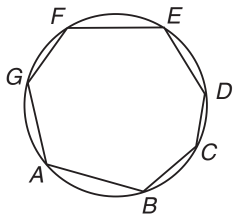
Thus, it is clear that a person sings only with a person who is located diagonally opposite to him.

$\therefore$ Total Singing Time = number of combinations × time required by one combination

$$28 = \frac{N(N-3)}{2} \times 2$$

$\Rightarrow$ $N = 7$

Following are the possible pairs of singers

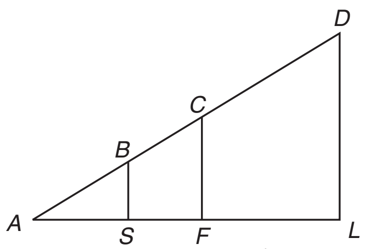


AC, AD, AE, AF
BD, BE, BF, BG
CE, CF, CG
DF, DG
EG

13. A nuts producing machine produces 1000 nuts in a single stretch of 10 minutes and then it stops for 5 minutes. Thus, to produce 9000 nuts, this machine has to work for 9 stretches and halt for 8 intervals of 5 minutes each.
Thus, the required time to produce 9000 nuts

$$= 10 \times 9 + 5 \times 8 = 130 \text{ minutes}$$

14. Let BS, CF, DL be the son, father and lamp respectively.



where $BS = 0.9$ m, $CF = 1.8$ m and $DL = 6$ m
By the concept of similarity of triangles

$$\frac{BS}{AS} = \frac{CF}{AF} = \frac{DL}{AL} = \frac{0.9}{AS} = \frac{1.8}{AF} = \frac{6}{AL}$$

$\Rightarrow$ $AF = 2AS$

Let $AS = x$, then $AF = 2x$ $\Rightarrow$ $SF = x$

$\therefore$ $$\frac{CF}{AF} = \frac{DL}{AL}$$

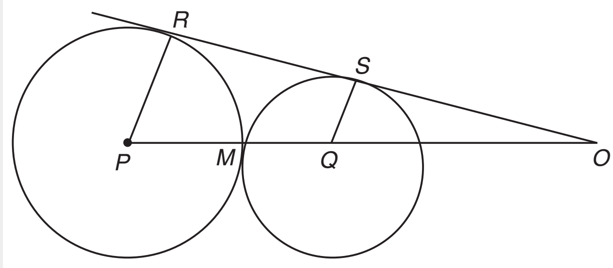
$\Rightarrow$ $$\frac{1.8}{2x} = \frac{6}{2x + 2.1}$$

$\Rightarrow$ $1.8(2x + 2.1) = 6 \times 2x$

$\Rightarrow$ $$x = \frac{9}{20} = 0.45 \text{ m}$$

Hence, the distance between son and father *i.e.*, $SF = 0.45$ m

✍ ***Solutions for question number 15-17 :*** *Since R & S are the points of contact of tangents, hence $\angle PRO$ and $\angle QSO$ are right angles. Also ΔPRO and ΔQSO are similar triangles.*



$\therefore$ $$\frac{PR}{QS} = \frac{PO}{QO}$$

$\Rightarrow$ $$\frac{4}{3} = \frac{PO}{QO} = \frac{PQ + QO}{QO}$$

$\Rightarrow$ $4(QO) = 3(PQ + QO)$

$\Rightarrow$ $$\frac{1}{3} = \frac{PQ}{QO}$$

15. $$\frac{PQ}{OQ} = \frac{1}{3}$$

Hence, (b) is correct.

16. Given $PQ + QO = 28$ cm $(\because PO = PQ + QO)$

Also, $\frac{PQ}{QO} = \frac{x}{3x}$ $\Rightarrow$ $x + 3x = 28$ cm

$\therefore$ $PQ = 7$ cm

But $PQ = PM + MQ$; $MQ = SQ$ is the radius of circle II

$$7 = 4y + 3y$$

$\Rightarrow$ $MQ = 3$ cm

Hence, the radius of circle II is 3 cm

17. In ΔQSO, $SO = \sqrt{(QO)^2 - (QS)^2}$ ($\because \Delta QSO$ is a right angled)

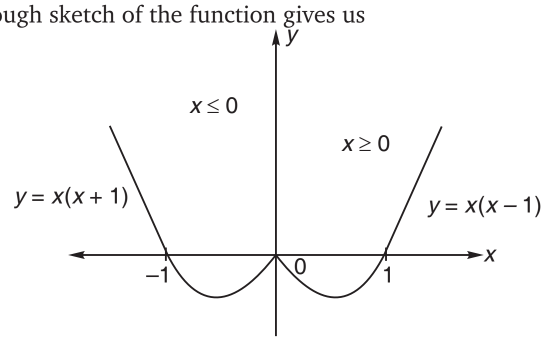
$$= \sqrt{(21)^2 - (3)^2} = \sqrt{432} = 12\sqrt{3} \text{ cm}$$

18. $f(x) = ax^2 - b|x|$

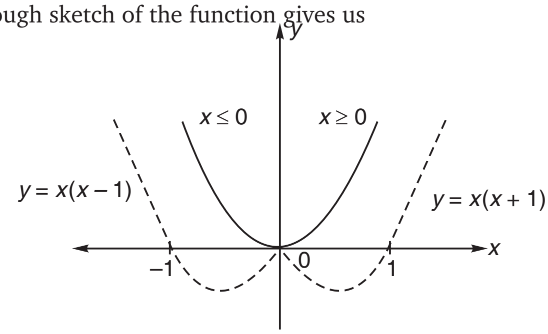
x	$f(x)$; when $a > 0$, $b > 0$	$f(x)$; when $a > 0$, $b < 0$
-2	$4a - 2b$	$4a + 2b$
-1	$a - b$	$a + b$
$-\frac{1}{2}$	$\frac{a}{4} - \frac{b}{2}$	$\frac{a}{4} + \frac{b}{2}$
0	0	0
$\frac{1}{2}$	$\frac{a}{4} - \frac{b}{2}$	$\frac{a}{4} + \frac{b}{2}$
1	$a - b$	$a + b$
2	$4a - 2b$	$4a + 2b$

For $a > 0$, $b > 0$, at $x = 0$ $f(x)$ is neither maximum nor minimum.
Hence choice (a) and (c) are eliminated.
For $a > 0$, $b < 0$, at $x = 0$ $f(x)$ is minimum.
Hence, choice (d) is correct.
Alternatively : $f(x) = y = ax^2 + b|x|$
As per the choices (a) and (c), $a > 0$, $b > 0$
we can assume $a = b = 1$

Therefore, the equation reduces to $y = x^2 - |x|$ and hence a rough sketch of the function gives us



So, at $x = 0$, we neither have a maxima nor a minima.
As per the choices (b) and (d) $a > 0$, $b < 0$, we can assume $a = 1$ and $b = -1$
Therefore, the equation reduces to $y = x^2 + |x|$ and hence a rough sketch of the function gives us



So, at $x = 0$, we have a minima.
Hence, choice (d) is correct.

19. Let A, B, G, F denote respectively number of adults, boys, girls and families.

Therefore, $$A > B > G > F$$

Now, go through options and consider the least possible value for number of families.
Hence, consider choice (c) *i.e.*, $F = 2$

$\therefore$ $$F < G < B < A$$
$\Rightarrow$ $$2 < 3 < 4 < 5$$

This shows that there are more than 2 adults in any one of the two families which is impossible.
Hence, choice (c) is wrong.
Now, we consider choice (d) *i.e.*, $F = 3$

$\therefore$ $$F < G < B < A$$
$\Rightarrow$ $$3 < 4 < 5 < 6$$

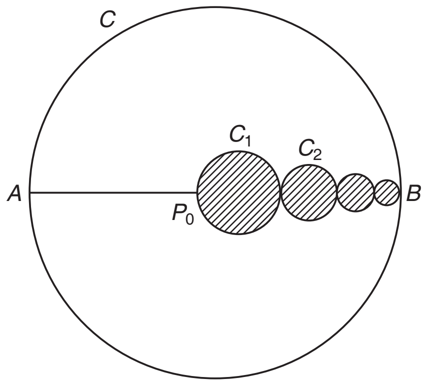
This shows that each family has at most two adults in each of the 3 families, which is the required condition. Also, it shows that there are atleast 3 children in each of the 3 families, which is the required condition. Since, all the required conditions are satisfied for the minimum 3 families.
Hence, choice (d) is correct.

20. Given $x + y = xy$

$\Rightarrow$ $$xy - x - y = 0$$
$\Rightarrow$ $$xy - x - y + 1 = 1 \quad \text{(adding 1 to both sides)}$$
$\Rightarrow$ $$y(x-1) - 1(x-1) = 1$$
$\Rightarrow$ $$(x-1)(y-1) = 1 \quad \ldots\text{(i)}$$

As x and y are integers, $(x-1)$ and $(y-1)$ are integers. Hence $(x-1)$ and $(y-1)$ must both be 1 or -1 to satisfy the equation (i).
Hence, $x = 2$, $y = 2$ or $x = 0$, $y = 0$
Thus, only two integer pairs for (x, y) satisfy the condition $x + y = xy$.

21.



Let the diameters of circles $C, C_1, C_2, C_3, \ldots$ etc. are $AB, \frac{AB}{4}, \frac{AB}{8}, \frac{AB}{16} \ldots$ etc.
Let radius of circle C be R then the radii of circles $C_1, C_2, C_3, \ldots$ etc. be $\frac{R}{4}, \frac{R}{8}, \frac{R}{16} \ldots$ etc.

$\therefore$ $$\text{Area of circle} = \pi R^2 = S \text{ (say)}$$

Sum of areas of circles $C_1, C_2, C_3, \ldots$ etc.

$$= \pi R^2 \left[\frac{1}{16} + \frac{1}{64} + \frac{1}{256} + \ldots\right]$$

$$= \pi R^2 \left[\frac{\frac{1}{16}}{1 - \frac{1}{4}}\right] = \frac{\pi R^2}{12} = \frac{S}{12}$$

$\therefore$ $$\text{Area of shaded region} = \frac{S}{12}$$

Hence, the area of unshaded region $= S - \frac{S}{12} = \frac{11S}{12}$

$\therefore$ $$\frac{\text{area of unshaded region}}{\text{area of circle } C} = \frac{\frac{11S}{12}}{S} = \frac{11}{12}$$

22. The terms of the given sequence are as follows :

$a_1 = 81.33 \rightarrow (a_1)$ $\quad a_7 = a_1$
$a_2 = -19 \rightarrow (a_2)$ $\quad a_8 = a_2$
$a_3 = -100.33 \rightarrow (a_3)$ $\quad a_9 = a_3$
$a_4 = -81.33 \rightarrow (-a_1)$ $\quad a_{10} = -a_1$
$a_5 = 19 \rightarrow (-a_2)$ $\quad a_{11} = -a_2$
$a_6 = 100.33 \rightarrow (-a_3)$ $\quad a_{12} = -a_3$ etc.

From the above pattern we can conclude that the sum of every six consecutive terms (first term onwards) is zero. Thus the sum of first 6000 terms is zero.
Also from the above pattern it is clear that
6001st term $= a_1$ and 6002nd term $= a_2$
$\therefore$ Sum of first 6002 term = sum of first 6000 term $+ a_1 + a_2$

$$= 0 + 81.33 - 19 = 62.33$$

23. For convenience, let us assume radius of the circle $r = 1$ m.
Then the circumference of the circle $= 2\pi r = 2\pi$ metre
Hence, the sprinter has to cover $2\pi m$ in each round but we know that speed × time = distance
The table below shows the distance traversed in different successive time periods with different (given) speeds.

Time	Speed	Distance
$\frac{1}{2}$ min	π m/min	$\frac{\pi}{2}$ m
1 min	$\frac{\pi}{2}$ m/min	$\frac{\pi}{2}$ m
2 min	$\frac{\pi}{4}$ m/min	$\frac{\pi}{2}$ m
4 min	$\frac{\pi}{8}$ m/min	$\frac{\pi}{2}$ m
8 min	$\frac{\pi}{16}$ m/min	$\frac{\pi}{2}$ m
16 min	$\frac{\pi}{32}$ m/min	$\frac{\pi}{2}$ m
32 min	$\frac{\pi}{64}$ m/min	$\frac{\pi}{2}$ m
64 min	$\frac{\pi}{128}$ m/min	$\frac{\pi}{2}$ m

From the table it is clear that in each successive time duration the distance traversed is same.
Also, we can say that to cover 2π distance (*i.e.*, one round of the circular path) sprinter needs 4 time intervals for each round.
$\therefore$ Time required to cover first round, second round, third round etc. is $\frac{15}{2}$ minutes, 120 minutes, 1920 minutes etc.
Hence, the ratio of time taken for the nth round to that for the previous round is 16, since in each next round sprinter requires 16 times the time required in the previous round.

24. We have, $u = (\log_2 x)^2 - 6\log_2 x + 12$

Let $\log_2 x = p$ ...(1)

$\therefore$ $u = p^2 - 6p + 12$

Also, we have $x^4 = 256 = 2^8$

$\Rightarrow$ $\log_2 x^4 = \log_2 2^8$ {Applying log to base 2 on both sides}

$\Rightarrow$ $4\log_2 x = 8$ ...(2)

Dividing (2) by (1) we get

$$u = \frac{8}{p}$$

$$\Rightarrow \frac{8}{p} = p^2 - 6p + 12$$

$$\Rightarrow p^3 - 6p^2 + 12p - 8 = 0$$

$$\Rightarrow (p-2)^3 = 0$$

$$\Rightarrow p = 2$$

$$\Rightarrow \log_2 x = 2$$

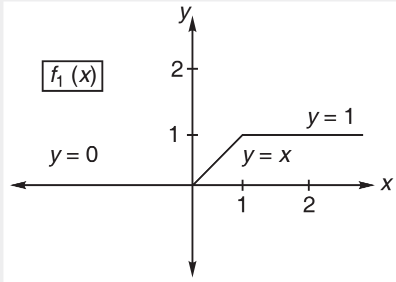
$$\Rightarrow x = 2^2 = 4$$

Thus, we have exactly one solution.

✍ ***Solutions for question number 25 and 26 :*** *Consider the function*

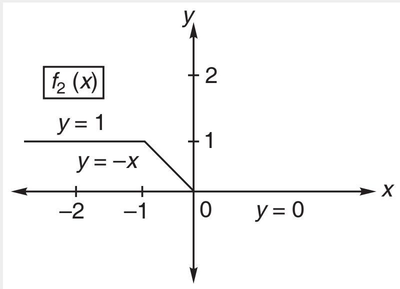
$$f_1(x) = \begin{cases} x & \text{for } 0 \le x \le 1 \\ 1 & \text{for } x \ge 1 \\ 0 & \text{for all other values of } x \end{cases}$$

The graph of the function $f_1(x)$ is given below.



The graph of the function $f_2(x)$ is given below, which is obtained by reflecting $f_1(x)$ in y-axis.
Since $f_2(x) = f_1(-x)$

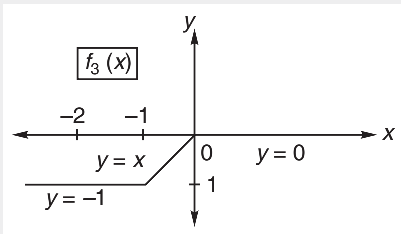
NOTE Replacing x with $(-x)$ means reflecting the graph in y-axis.



The graph of the function $f_3(x)$ is given below, which is obtained by reflecting $f_2(x)$ in x-axis.
Since, $f_3(x) = -f_2(x)$

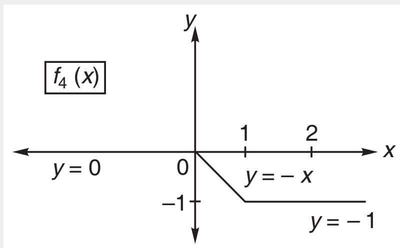
NOTE Introduction of – sign means reflecting in x-axis.

The graph of the function $f_4(x)$ is given below, which is obtained by reflecting $f_3(x)$ in y-axis.



Since, $f_4(x) = f_3(-x)$

25. Consider the product $f_1(x)\, f_2(x)$;



for $x \ge 0$, $f_2(x) = 0$, hence $f_1(x)\, f_2(x) = 0$
and for $x < 0$, $f_1(x) = 0$, hence $f_1(x)\, f_2(x) = 0$
Thus, for all values of x, $f_1(x)\, f_2(x) = 0$
Consider the product $f_2(x)\, f_3(x)$;

for $x \geq 0$, $f_2(x) = 0 = f_3(x)$, hence $f_2(x)\, f_3(x) = 0$

and for $x < 0$, $f_2(x) > 0$ and $f_3(x) < 0$, hence $f_2(x)\, f_3(x) < 0$

Thus, for all values of x, $f_1(x)\, f_2(x) \neq 0$;

Consider the product $f_2(x)\, f_4(x)$;

for $x \geq 0$, $f_2(x) = 0$, hence $f_2(x)\, f_4(x) = 0$

for $x < 0$, $f_4(x) = 0$, hence $f_2(x)\, f_4(x) = 0$

Thus, for all values of x, $f_2(x)\, f_4(x) = 0$

Hence, choice (c) is correct.

26. **Choice (a) :** From the graphs it can be observed that $f_1(x) = f_4(x)$ for $x \leq 0$ but $f_1(x) \neq f_4(x)$ for $x > 0$.
Hence, choice (a) is ruled out.
Choice (b) : To obtain the graph of $-f_3(-x)$ the graph of $f_4(x)$ is to be reflected in x-axis, this would give the graph of $f_1(x)$.
Hence, $f_1(x) = -f_3(-x)$ is true for all values of x.
Choice (c) : The graph of $f_2(-x)$ is obtained by the reflection of the graph of $f_2(x)$ in y-axis, which gives us the graph of $f_1(x)$ and not $f_4(x)$.
Hence, choice (c) is ruled out
Choice (d) : for $x < 0$ $f_1(x) = 0$ and $f_3(x) < 0$

hence, $\quad f_1(x) + f_3(x) < 0$

for $x > 0$, $\quad f_1(x) > 0$ and $f_3(x) = 0$

hence, $\quad f_1(x) + f_3(x) > 0$

Therefore, choice (d) is also ruled out.

27. Let the each side (or edge) of the cube be 'a', then the diagonal of the cube is '$a\sqrt{3}$' *i. e.*, $DF = AG = CE = a\sqrt{3}$
$\therefore$ Each side of the equilateral triangle is $a\sqrt{3}$

and the altitude of this triangle is $a\sqrt{3} \times \dfrac{\sqrt{3}}{2} = \dfrac{3}{2}a$

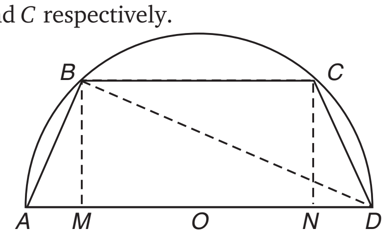
$\therefore$ the circumradius of this triangle $= \dfrac{3}{2}a \times \dfrac{2}{3} = a$

(Since centroid divides the altitude in the ratio of 2 : 1)
Hence, the radius of the circumscribing circle is equal to the side of the cube.

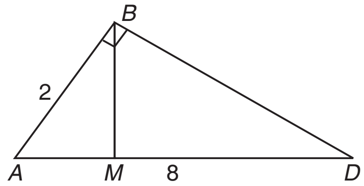
28. There are 3 segments in the northward routes and 5 segments in the westward routes. Thus there are total $3 + 5 = 8$ segments in any one of the routes from A to B. Therefore the car can move only any 3 segments to the north and any 5 segments to the west.
The number of distinct routes is equal to the number of ways of selecting 3 segments out of the 8 segments along which the car can go north and selecting 5 segments out of the remaining 5 segments along which the car can go west
Therefore the number of distinct routes from A to B is ${}^8C_3 \times {}^5C_5 = 56$

NOTE Another possibility is ${}^8C_5 \times {}^3C_3 = 56$, which is the same result as obtained above.

29. Draw the perpendiculars BM and CN on the diameter AD from points B and C respectively.



Now join the points B and D.
Then the $\Delta\, ABD$ is a right angled triangle as $\angle ABD = 90°$ (angle in a semicircle is a right angle)



In $\Delta\, ABD$,

$$AM = \frac{AB^2}{AD} = \frac{4}{8} = \frac{1}{2}$$

Similarly,

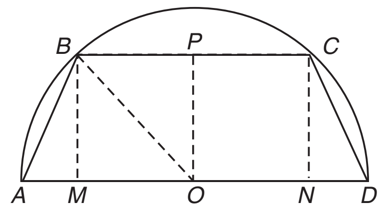
$$ND = AM = \frac{1}{2}$$

$$\therefore \quad AD = AM + MN + ND$$

$$8 = \frac{1}{2} + MN + \frac{1}{2} \Rightarrow MN = 7$$

$$\because \quad BC = MN \quad \therefore BC = 7$$

Alternatively : Draw the perpendiculars BM, OP and CN as shown in figure.



$$BO = AO = \text{radius} = 4$$

$$\therefore \quad \text{Area of } \Delta\, ABO = \frac{1}{2} AO \times BM$$

$$= \frac{1}{2} \times 4 \times BM = 2BM \quad \ldots(1)$$

$$\text{Semiperimeter of } \Delta\, ABO = s = \frac{2 + 4 + 4}{2} = 5$$

$$\therefore \quad \text{Area of } \Delta\, ABO = \sqrt{s(s-a)(s-b)(s-c)}$$

$$= \sqrt{5(5-2)(5-4)(5-4)}$$

$$= \sqrt{15} \quad \ldots(2)$$

From (1) and (2), $2BM = \sqrt{15}$

$$\Rightarrow \quad BM = \frac{\sqrt{15}}{2} = OP = CN$$

$$\therefore \quad BP = \sqrt{(BO)^2 - (OP)^2} = \sqrt{16 - \frac{15}{4}} = \frac{7}{2}$$

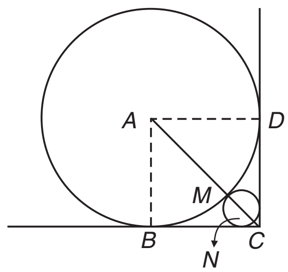
$$\therefore \quad BC = BP + PC = \frac{7}{2} + \frac{7}{2} = 7 \; (\because \; BP = PC = MO = ON)$$

30. Let the radii of the larger and smaller circles be respectively R and r.
$\therefore$ In the figure

$$AB = AD = AM = R = 2$$

and $\quad MN = r$

A and N are centres of the larger and the smaller circles, respectively and the two circles touch each other at M.
Since, $\angle BCD$ is 90° and B and D are point of contact of the tangents
$\therefore$ $ABCD$ is a square.



$$\therefore \quad AC = \sqrt{2}R$$

Similarly, $\quad CN = \sqrt{2}\, r$

But

$$AC = AM + MN + NC$$

$$R\sqrt{2} = R + r + r\sqrt{2}$$

$\Rightarrow \quad R(\sqrt{2} - 1) = r(\sqrt{2} + 1)$

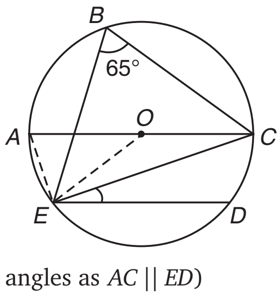
$\Rightarrow \quad r = \dfrac{R(\sqrt{2} - 1)}{(\sqrt{2} + 1)} = \dfrac{R(\sqrt{2} - 1)}{(\sqrt{2} + 1)} \times \dfrac{(\sqrt{2} - 1)}{(\sqrt{2} - 1)}$

$\Rightarrow \quad r = R(3 - 2\sqrt{2})$

$\Rightarrow \quad r = 2(3 - 2\sqrt{2}) = 6 - 4\sqrt{2} \qquad (\because R = 2)$

31. $\angle EAC = \angle EBC = 65°$
($\because$ angles in the same segment are equal)

$\angle AEC = 90°$
(angle in the semicircle)



$\therefore$ In ΔAEC,

$\angle EAC + \angle AEC + \angle ACE = 180°$

$65° + 90° + \angle ACE = 180°$

$\Rightarrow \quad \angle ACE = 25°$

$\therefore \quad \angle DEC = \angle ACE = 25°$

($\because \angle DEC$ and $\angle ACE$ are alternate angles as $AC \parallel ED$)

Alternatively : $\angle EOC = 2\angle EBC = 130°$

$\because \quad \angle OEC = \angle OCE \qquad (OE = OC = \text{radii})$

$\therefore \quad \angle OCE = \dfrac{180 - 130}{2} = 25°$

$\therefore \quad \angle CED = \angle OCE = \angle ACE = 25°$

✍ ***Solutions for question number 32 and 33 :*** *Let a, b, c be the number of questions in groups A, B and C respectively and the respective marks of the groups A, B and C be a, $2b$ and $3c$ respectively.*
Since, there are total 100 questions in all the three groups i.e., $a + b + c = 100$

32. Following table reveals the information about probable number of questions and the marks of each 5 group.

No. of Questions in			Marks of the Groups			Total Marks	% Marks of Group A
A	**B**	**C**	**A**	**B**	**C**		
a	b	c	a	$2b$	$3c$	$a + 2b + 3c$	$\dfrac{a}{a + 2b + 3c} \times 100$
76	23	1	76	46	3	125	$\dfrac{76}{125} > 60\%$
75	23	2	75	46	6	127	$\dfrac{75}{127} < 60\%$
74	23	3	74	46	9	129	$\dfrac{74}{129} < 60\%$
73	23	4	73	46	12	131	$\dfrac{73}{131} < 60\%$

From the above table it can be concluded that as the number of question in group c increases (or the number of questions in group A is decreases) the percentage marks of the group A decreases and goes below 60%.

Thus, there is only one possible combination of questions in which group A must have at least 60% marks . Hence, the number of questions in group c is 1.

NOTE We need not to calculate further since four values of c in the table shows the possible pattern of percentage marks for group A).

Best way is to go through options.

33. Group C contains exactly 8 questions, therefore carry 24 marks. But group A carries atleast 60% marks and group B carries atleast 20% marks, it means group C carries atmost 20% marks with 24 marks. Hence the minimum marks (aggregate of all the three groups A, B and C) must be 120.
The following table reveals the information regarding number of questions and respective marks of each group.

No. of Questions in Groups			Marks in Groups			Total Score	%Marks of Group A	% Marks of Group B	Condition Satisfied If $x \geq 60\%$ and $y \geq 20\%$
C	**B**	**A**	**C**	**B**	**A**		(x)	(y)	
c	b	a	$3c$	$2b$	a	$a + 2b + 3c$	$= \dfrac{a}{a + 2b + 3c} \times 100$	$= \dfrac{2b}{a + 2b + 3c} \times 100$	
8	11	81	24	22	81	127	$x > 60\%$	$y < 20\%$	✗
8	12	80	24	24	80	128	$x > 60\%$	$y < 20\%$	✗
8	13	79	24	26	79	129	$x > 60\%$	$y > 20\%$	✓
8	14	78	24	28	78	130	$x = 60\%$	$y > 20\%$	✓
8	15	77	24	30	77	131	$x < 60\%$	$y > 20\%$	✗

From the above table we can observe that there are only two possible values of b *viz.*, 13 and 14

i.e., $\quad b = 13$ or $b = 14$.

Hence, choice (c) is correct.

Alternatively : Best way is to go through options to save the calculation time.

34. $a^n + b^n$ is always divisible by $a + b$ when n is odd.

$\therefore 15^{23} + 23^{23}$ is always divisible by $15 + 23 = 38$.

As the given expression is divisible by 38, so it must be divisible by all the factors of 38 and hence by 19.
Thus, we get a remainder 0, since $(15^{23} + 23^{23})$ is divisible by 19.

35. Any of the four colours can be chosen for the first stripe. Any of the remaining three colours can be chosen for the second stripe. The third stripe can again be coloured in 3 ways (since we can repeat the colour of the first stripe but can not use the colour of second stripe).
Similarly, there are 3 ways to colour each of the remaining stripes (since the colour which has been used in the previous stripe can not be used in the next stripe, but other colours can be used)
$\therefore$ The number of ways the flag can be coloured is

$$4 \times (3)^5 = 12 \times 3^4 = 12 \times 81$$

❒❒❒