CAT SOLVED PAPER

No. of Questions : 30 *Time :* 40 *min*

[Section : A]

NOTE Questions 1 to 10 carry one mark each. There is $\frac{1}{3}$ negative mark for each wrong answer.

✍ ***Directions for questions 1 to 8 :*** *Answer the questions independently of each other.*

1. If $R = \frac{30^{65} - 29^{65}}{30^{64} + 29^{64}}$, then :

(a) $0 < R < 0.1$ (b) $0.1 < R \le 0.5$
(c) $0.5 < R \le 1.0$ (d) $R > 1.0$

2. What is the distance in cm between two parallel chords of lengths 32 cm and 24 cm in a circle of radius 20 cm ?

(a) 1 or 7 (b) 2 or 14
(c) 3 or 21 (d) 4 or 28

3. For which value of k does the following pair of equations yield a unique solution for x such that the solution is positive ?

$$x^2 - y^2 = 0$$

$$(x - k)^2 + y^2 = 1$$

(a) 2 (b) 0
(c) $\sqrt{2}$ (d) $-\sqrt{2}$

4. If $x = (16^3 + 17^3 + 18^3 + 19^3)$, then x divided by 70 leaves a remainder of :

(a) 0 (b) 1
(c) 69 (d) 35

5. A chemical plant has four tanks (A, B, C and D), each containing 1000 litres of a chemical. The chemical is being pumped from one tank to another as follows :

From A to B @ 20 litres/min
From C to A @ 90 litres/min
From A to D @ 10 litres/min
From C to D @ 50 litres/min
From B to C @ 100 litres/min
From D to B @ 110 litres/min

Which tank gets emptied first, and how long does it take (in minutes) to get empty after pumping starts ?

(a) A, 16.66 (b) C, 20
(c) D, 20 (d) D, 25

6. Two identical circles intersect so that their centres, and the points at which they intersect, form a square of side 1 cm. The area in sq. cm of the portion that is common to the two circles is:

(a) $\frac{\pi}{4}$ (b) $\frac{\pi}{2} - 1$
(c) $\frac{\pi}{5}$ (d) $\sqrt{2} - 1$

7. A jogging park has two identical circular tracks touching each other, and a rectangular track enclosing the two circles. The edges of the rectangles are tangential to the circles. Two friends, A and B, start jogging simultaneously from the point where one of the circular tracks touches the smaller side of the rectangular track, A jogs along the rectangular track, while B jogs along the two circular tracks in a figure of eight. Approximately, how much faster than A does B have to run, so that they take the same time to return to their starting point ?

(a) 3.88% (b) 4.22%
(c) 4.44% (d) 4.72%

8. In a chess competition involving some boys and girls of a school, every student had to play exactly one game with every other student. It was found that in 45 games both the players were girls, and in 190 games both were boys. The number of games in which one player was a boy and the other was a girl is :

(a) 200 (b) 216
(c) 235 (d) 256

✍ ***Directions for questions 9 and 10 :*** *Answer the questions on the basis of the information given below.*
Ram and Shyam run a race between points A and B, 5 km apart. Ram starts at 9 a.m. from A at a speed of 5 km/h, reaches B, and returns to A at the same speed. Shyam starts at 9.45 a.m. from A at a speed of 10 km/h, reaches B and comes back to A at the same speed.

9. At what time do Ram and Shyam first meet each other ?

(a) 10 a.m. (b) 10:10 a.m.
(c) 10:20 a.m. (d) 10:30 a. m.

10. At what time does Shyam overtake Ram ?

(a) 10:20 a.m. (b) 10:30 a.m.
(c) 10:40 a.m. (d) 10:50 a.m.

[Section : B]

NOTE Questions 11 to 30 carry two marks each. There is $\frac{2}{3}$ negative mark for each wrong answer.

✍ ***Directions for questions 11 to 30 :*** *Answer the questions independently of each other.*

11. Let $x = \sqrt{4 + \sqrt{4 - \sqrt{4 + \sqrt{4 - \dots \text{to infinity}}}}}$. Then x equals :

(a) 3 (b) $\left(\frac{\sqrt{13} - 1}{2}\right)$
(c) $\left(\frac{\sqrt{13} + 1}{2}\right)$ (d) $\sqrt{13}$

12. Let $g(x)$ be a function such that $g(x + 1) + g(x - 1) = g(x)$ for every real x. Then for what value of p is the relation $g(x + p) = g(x)$ necessarily true for every real x ?

(a) 5 (b) 3
(c) 2 (d) 6

13. A telecom service provider engages male and female operators for answering 1000 calls per day. A male operator can handle 40 calls per day whereas a female operator can handle 50 calls per day. The male and the female operators get a fixed wage of Rs. 250 and Rs. 300 per day respectively. In addition, a male operator gets Rs. 15 per call he answers and a female operator gets Rs. 10 per call she answers. To minimize the total cost, how many male operators should the service provider employ assuming he has to employ more than 7 of the 12 female operators available for the job ?

(a) 15 (b) 14
(c) 12 (d) 10

14. Three Englishmen and three Frenchmen work for the same company. Each of them knows a secret not known to others. They need to exchange these secrets over person-to-person phone calls so that eventually each person knows all six secrets. None of the Frenchmen knows English, and only one Englishman knows French. What is the minimum number of phone calls needed for the above purpose ?

(a) 5 (b) 10
(c) 9 (d) 15

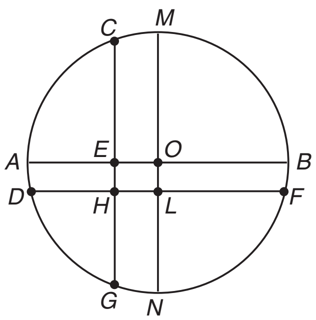
15. A rectangular floor is fully covered with square tiles of identical size. The tiles on the edges are white and the tiles in the interior are red. The number of white tiles is the same as the number of red tiles. A possible value of the number of tiles along one edge of the floor is :

(a) 10 (b) 12
(c) 14 (d) 16

16. Let $n! = 1 \times 2 \times 3 \times \ldots \times n$ for integer $n \geq 1$. If $p = 1! + (2 \times 2!) + (3 \times 3!) + \ldots + (10 \times 10!)$, then $p + 2$ when divided by $11!$ leaves a remainder of :

(a) 10 (b) 0
(c) 7 (d) 1

17. In the following figure, the diameter of the circle is 3 cm. AB and MN are two diameters such that MN is perpendicular to AB. In addition CG is perpendicular to AB such that $AE : EB = 1 : 2$, and DF is perpendicular to MN such that $NL : LM = 1 : 2$. The length of DH in cm is :



(a) $2\sqrt{2} - 1$ (b) $\frac{(2\sqrt{2} - 1)}{2}$
(c) $\frac{(3\sqrt{2} - 1)}{2}$ (d) $\frac{(2\sqrt{2} - 1)}{3}$

18. The digits of a three-digit number A are written in the reverse order to form another three-digit number B. If $B > A$ and $B - A$ is perfectly divisible by 7, then which of the following is necessarily true ?

(a) $100 < A < 299$ (b) $106 < A < 305$
(c) $112 < A < 311$ (d) $118 < A < 317$

19. If $a_1 = 1$ and $a_{n+1} - 3a_n + 2 = 4n$ for every positive integer n, then a_{100} equals :

(a) $3^{99} - 200$ (b) $3^{99} + 200$
(c) $3^{100} - 200$ (d) $3^{100} + 200$

20. Let S be the set of five-digit numbers formed by the digits 1, 2, 3, 4 and 5, using each digit exactly once such that exactly two odd positions are occupied by odd digits. What is the sum of the digits in the rightmost position of the numbers in S ?

(a) 228 (b) 216
(c) 294 (d) 192

21. The rightmost non-zero digit of the number 30^{2720} is :

(a) 1 (b) 3
(c) 7 (d) 9

22. Four points A, B, C and D lie on a straight line in the X-Y plane, such that $AB = BC = CD$, and the length of AB is 1 metre. An ant at A wants to reach a sugar particle at D. But there are insect repellents kept at points B and C. The ant would not go within one metre of any insect repellent. The minimum distance in metres the ant must traverse to reach the sugar particle is :

(a) $3\sqrt{2}$ (b) $1 + \pi$
(c) $\frac{4\pi}{3}$ (d) 5

23. If $x \geq y$ and $y > 1$ then the value of the expression $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$ can never be :

(a) – 1 (b) – 0.5
(c) 0 (d) 1

24. For a positive integer n, let p_n denote the product of the digits of n, and S_n denote the sum of the digits of n. The number of integers between 10 and 1000 for which $p_n + S_n = n$ is :

(a) 81 (b) 16
(c) 18 (d) 9

25. Rectangular tiles each of size 70 cm $\times$ 30 cm must be laid horizontally on a rectangular floor of size 110 cm by 130 cm, such that the tiles do not overlap. A tile can be placed in any orientation so long as its edges are parallel to the edges of the floor. No tile should overshoot any edge of the floor. The maximum number of tiles that can be accommodated on the floor is :

(a) 4 (b) 5
(c) 6 (d) 7

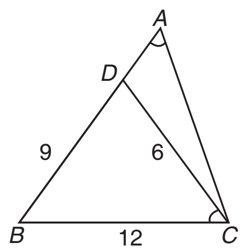
26. In the X-Y plane, the area of the region bounded by the graph of $|x + y| + |x - y| = 4$ is :

(a) 8 (b) 12
(c) 16 (d) 20

27. Consider a triangle drawn on the X-Y plane with its three vertices at (41, 0), (0, 41) and (0, 0), each vertex being represented by its (X, Y) coordinates. The number of points with integer coordinates inside the triangle (excluding all the points on the boundary) is :

(a) 780 (b) 800
(c) 820 (d) 741

28. Consider the triangle ABC shown in the following figure where $BC = 12$ cm, $DB = 9$ cm, $CD = 6$ cm and $\angle BCD = \angle BAC$.



What is the ratio of the triangle ADC to that of the triangle BDC ?

(a) $\frac{7}{9}$ (b) $\frac{8}{9}$
(c) $\frac{6}{9}$ (d) $\frac{5}{9}$

29. P, Q, S, R are points on the circumference of a circle of radius r, such that PQR is an equilateral triangle and PS is a diameter of the circle. What is the perimeter of the quadrilateral $PQSR$?

(a) $2r(1+\sqrt{3})$ (b) $2r(2+\sqrt{3})$
(c) $r(1+\sqrt{5})$ (d) $2r+\sqrt{3}$

30. Let S be a set of positive integers such that every element n of S satisfies the conditions

(1) $1000 \le n \le 1200$ (2) every digit in n is odd

Then how many elements of S are divisible by 3 ?

(a) 9 (b) 10
(c) 11 (d) 12

Answers

1 (d)	**2.** (d)	**3.** (c)	**4.** (a)	**5.** (c)	**6.** (b)	**7.** (d)	**8.** (a)	**9.** (b)	**10.** (b)
11. (c)	**12.** (d)	**13.** (d)	**14.** (c)	**15.** (b)	**16.** (d)	**17.** (b)	**18.** (b)	**19.** (c)	**20.** (b)
21. (a)	**22.** (b)	**23.** (d)	**24.** (d)	**25.** (c)	**26.** (c)	**27.** (a)	**28.** (a)	**29.** (a)	**30.** (a)

Hints & Solutions

1. $\dfrac{30^2-29^2}{30+29}=1, \dfrac{30^3-29^3}{30^2+29^2}\approx 1.5, \dfrac{30^4-29^4}{30^3+29^3}\approx 2\ldots$ etc.

So, it is clear that $\dfrac{30^{65}-29^{65}}{30^4+29^{64}}>1$

Alternatively : Since a^n-b^n

$$=(a-b)(a^{n-1}+a^{n-2}b+a^{n-3}b^2+\ldots+b^{n-1})$$

$$\therefore \frac{30^{65}-29^{65}}{30^{64}+29^{64}}$$

$$=\frac{(30-29)(30^{64}+30^{63}\times 29+30^{62}\times 29^2+\ldots+29^{64})}{30^{64}+29^{64}}$$

$$=\frac{[30^{64}+29^{64}+(30^{63}\times 29+30^{62}\times 29^2+\ldots)]}{30^{64}+29^{64}}$$

$$=1+\frac{(30^{63}+29+30^{62}\times 29^2+\ldots 30\times 29^{63})}{30^{64}+29^{64}}$$

Hence $R>1$

Alternatively : Applying componendo and dividendo

$$\frac{30^{65}-29^{65}}{30^{64}+29^{64}}=\frac{(30^{65}-29^{65})+(30^{64}+29^{64})}{(30^{65}-29^{65})-(30^{64}+29^{64})}$$

$$=\frac{30^{64}(30+1)+29^{64}(1-29)}{30^{64}(30-1)-29^{64}(1+29)}$$

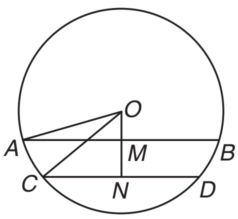
$$=\frac{30^{64}\times 31-29^{64}\times 28}{30^{64}\times 29-29^{64}\times 30}$$

Since numerator is greater than denominator.

Hence $R>1$

2. Case 1 : When both the chords are on the same side

Let O be the centre of circle and AB, CD be the two chords where $AB=32$ cm and $CD=24$ cm



$\therefore\ OM=\sqrt{(AO)^2-(AM)^2}$

$OM=\sqrt{(20)^2-(16)^2}$; $OA=20$ cm and $AM=16$ cm

$\Rightarrow \quad OM=12$ cm

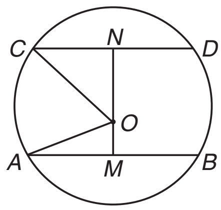
Similarly, $ON=\sqrt{(OC)^2-(CN)^2}$

$ON=16$ cm $\quad (\because\ OC=20$ cm and $CN=12$ cm$)$

HINT ΔOMA and ΔONC are right angled triangle, since OM and ON are perpendiculars drawn on AB and CD respectively. Also M and N are the mid points of AB and CD respectively.

Therefore $MN=ON-OM$

$=16-12=4$ cm



Case 2 : Here $OM=12$

and $ON=16$ cm

$\therefore \quad MN=12+16=28$ cm

Hence, choice (d) is correct.

3. $$x^2-y^2=0 \quad \ldots(1)$$

$$(x-k)^2+y^2=1 \quad \ldots(2)$$

From equation (1) $x^2=y^2$

$\therefore \quad (x-k)^2+x^2=1$

$\Rightarrow \quad x^2+k^2-2kx+x^2-1=0$

$\Rightarrow \quad 2x^2-2kx+(k^2-1)=0 \quad \ldots(3)$

This equation will give unique solution only when $D=0$ *i. e.,* $b^2-4ac=0 \Rightarrow b^2=4ac$

$\therefore \quad (-2k)^2=4\times 2\times(k^2-1)$

$\Rightarrow \quad k^2=2 \Rightarrow k=\pm\sqrt{2}$

For positive solution of x, $k=+\sqrt{2}$

Since at $k=-\sqrt{2}$, x becomes negative.

Hence, choice (c) is correct.

4. $\dfrac{16^3+17^3+18^3+19^3}{70}=\dfrac{16^3+19^3+17^3+18^3}{70}$

$(16+19)(16^2+19^2-16\times 19)$

$$= \frac{+ (17+18)(17^2+18^2-17\times 18)}{70}$$

$$= \frac{35\,(16^2+19^2-16\times 19+17^2+18^2-17\times 18)}{70}$$

$$= \frac{35\times 2m}{70} \qquad \text{(which is divisible by 70)}$$

hence remainder is zero.

Here $2m = (16^2+19^2-16\times 19+17^2+18^2-17\times 18)$

Alternatively : For those students who cannot make the above adjustment, they add up the cubes, of 16, 17, 18 and 19 then divide the sum of cubes of 16, 17, 18 and 19 by 70.

5.

Tank	Inflow/ min.	Outflow/ min	Net inflow/ min
A	90	10 + 20 = 30	90 – 30 = 60 litres
B	20 + 110 = 130	100	130 – 100 = 30 litres
C	100	90 + 50 = 140 litres	100 – 140 = – 40 litres
D	50 + 10 = 60	110	60 – 110 = – 50 litres

From the above table it is clear that the tank D is being emptied first. Since per min loss of chemical is maximum.

Hence required time $= \frac{1000}{50} = 20$ min

Alternatively : Go through options.

Let us consider choice (c)

$\therefore$ Total inflow of chemical in 20 min $= 1000 + 20(50+10)$

$= 2200$ litre

Total out flow of chemical in 20 min $= 20\times 110 = 2200$ litre

Hence in 20 min D gets emptied.

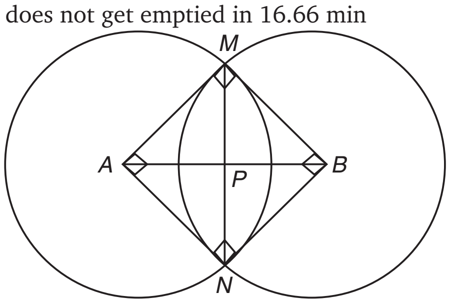
Hence we need not to check option (b) and (d).

Now, we can check option (a)

Total inflow 16.66 min $= 1000 + 20\times 90 = 2800$ litres

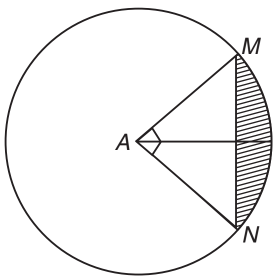
Total outflow in 16.66 min $= 16.66\times 30 = 500$ litre

Thus it does not get emptied in 16.66 min



Hence, choice (c) is correct.

6. $AM = BM = BN = AN = 1$ cm and $AMBN$ is a square.



The area of shaded region

= area of sector AMN

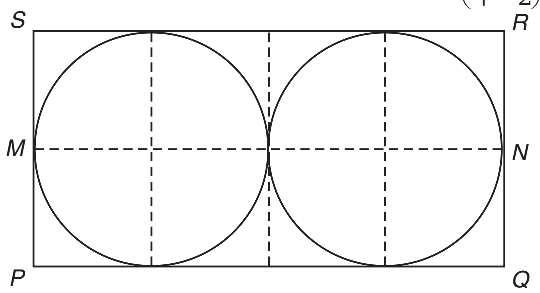
– area of $\Delta\, AMN$

$$= \pi\times(1)^2\times\frac{90}{360} - \frac{1\times 1}{2} = \frac{\pi}{4} - \frac{1}{2}$$

But since we have just equal area common inside the circle with centre B.

$\therefore$ Total area common to both the circles $= 2\left(\frac{\pi}{4}-\frac{1}{2}\right) = \frac{\pi}{2} - 1$

7.



Let the radius of each circle be 1 cm.

$\therefore$ Length of rectangle = 4 cm

and breath of rectangle = 2 cm

Thus the distance covered by A in one round

$$= 2\times 2\pi(1) = 4\pi = \frac{88}{7}\text{ cm}$$

and the distance covered by B in one round

$$= 2\,(4+2) = 12\text{ cm}$$

Remember, here time is constant, hence the speed is directly proportional to the distance.

$\therefore$ Difference in speed $= \frac{88}{7} - 12 = \frac{4}{7}$

hence percentage increase of speed of A over $B = \frac{\frac{4}{7}}{12}\times 100$

$= 4.76\%$

Hence, choice (d) is most appropriate.

8. Let B be the number of boys and G be the number of girls then

$${}^{B}C_2 = 190 \Rightarrow B = 20$$

and $${}^{G}C_2 = 45 \Rightarrow G = 10$$

$\therefore$ Total number of players $= 20 + 10 = 30$

$\therefore$ number of matches between a single boy and a single girl $= {}^{20}C_1\times{}^{10}C_1 = 200$

Alternatively : There can be only 3 cases

$B \leftrightarrow B$

$G \leftrightarrow G$

$B \leftrightarrow G$

$\therefore$ Required number of matches = total matches $- ({}^{B}C_2 + {}^{G}C_2)$

$= {}^{30}C_2 - ({}^{20}C_2 + {}^{10}C_2)$

$= 435 - (190 + 45) = 200$

✍ ***Solutions for question number 9 and 10 :***

Ram @ 5 km/h

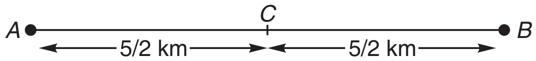
Shyam @ 10 km/h

Now, best way is to go through options

Ram @ 5 km/h reaches B at 10 am while Shyam @ 10 km/h reaches halfway (i.e., just between A and B) at 10 am. So Ram and Shyam does not meet each other at 10 am.

Again Shyam reaches at B at 10:15 am and at that time Ram was returning from B.

Hence Ram and Shyam must have been meet each other before 10:15 *am.*



9. Thus the correct choice is (b)

Alternatively :

Earlier we have observed that when Ram is at B, Shyam is at C at 10 am. The distance between B and C is $\frac{5}{2}$ km. and now since Ram and Shyam are moving toward each other (*i.e.,* towards C and B respectively). Therefore relative speed $= 10 + 5 = 15$ km/h

Therefore required time to meet each other

$$= \frac{\frac{5}{2}}{15} = \frac{1}{6} \text{ h} = 10 \text{ min.}$$

Hence they will meet each other at 10:10 am.

10. Now, go through suitable option.

Consider choice (b)

Total distance travelled by Ram in $1\frac{1}{2}$ half hours

$$= 5 \times \frac{3}{2} = \frac{15}{2} = 7.5 \text{ km}$$

Total distance travelled of Shyam in 45 minutes

$$= 10 \times \frac{45}{60} = 7.5 \text{ km}$$

Since both travelled the same distance till 10:30 am hence, it is clear that Shyam must overtake Ram at 10:30 am.

Alternatively : Since we know that Shyam is at B at 10:15 am. Therefore Ram is $\frac{5}{4}$ km away from B at 10:15 am.

Hence the required time by Shyam to overtake Ram

$$= \frac{\frac{5}{4}}{(10-5)} = \frac{1}{4} \text{ h} = 15 \text{ min.}$$

Since when two persons are moving in the same direction their relative speed becomes equal to the difference of their individual speeds.

Thus Shyam will overtake at 10:15 am + 15 min = 10 : 30 am

11.
$$x = \sqrt{4 + \sqrt{4 - \sqrt{4 + \sqrt{4 - \ldots \infty}}}}$$

$$\Rightarrow \quad x = \sqrt{4 + \sqrt{4 - x}}$$

$$\Rightarrow \quad x^2 = 4 + \sqrt{4 - x}$$

$$\Rightarrow \quad x^2 - 4 = \sqrt{4 - x}$$

$$\Rightarrow \quad (x^2 - 4)^2 = (4 - x) \quad \ldots(1)$$

Now go through options and find the suitable choice. The only appropriate choice is (c)

Since $\sqrt{13} \approx 3.6$, therefore $\frac{\sqrt{13}+1}{2} \approx 2.3$

Then substituting $x = 2.3$ in equation (1) we get the satisfactory result.

Note that other choices do not satisfy the equation (1).

12. $g(x + 1) + g(x - 1) = g(x)$

$$\Rightarrow \quad g(x + 1) = g(x) - g(x - 1)$$

Now, go through options.

Let us consider choice (d)

$$\therefore \quad g(x + 6) = g(x + 5) - g(x - 4)$$
$$= [g(x + 4) - g(x + 3)] - g(x + 4)$$
$$= - g(x + 3)$$
$$= - [g(x + 2) - g(x + 1)]$$
$$= - [g(x + 1) - g(x) - g(x + 1)]$$
$$= g(x)$$

Hence choice (d) is correct.

Alternatively : Let $g(x) = a$ and $g(x - 1) = b$

$$\therefore \quad g(x + 1) = g(x) - g(x - 1) = a - b$$
$$\therefore \quad g(x + 2) = g(x + 1) - g(x) = (a - b) - a = - b$$
$$g(x + 3) = g(x + 2) - g(x + 1) = - b - (a - b) = - a$$
$$g(x + 4) = g(x + 3) - g(x + 2) = - a - (- b) = b - a$$
$$g(x + 5) = g(x + 4) - g(x + 3) = (b - a) - (- a) = b$$
$$g(x + 6) = g(x + 5) - g(x + 4) = b - (b - a) = a$$

hence $\quad g(x + 6) = g(x) = a$

$$\therefore \quad p = 6$$

13. From the given data we can conclude that for the greater number of calls we have to deploy more female operators than male operators since the variable cost (*i. e.*, cost/call) is less for a female operator. Also a female operator answers more calls than a male operator.

Let us consider choice (a)

Number of male operators = 15

Number of calls answered by them = 15 × 40 = 600

Total cost = 15 × 250 + 600 × 15 = 12750

$\therefore$ Number of female operators $= \frac{(1000 - 600)}{50} = 8$

Number of calls answered by them = 8 × 50 = 400

Total cost = 8 × 300 + 400 × 10 = 6400

Total expenditure to answer 1000 calls = 12750 + 6400

= Rs. 19150

Please note that choices (b) and (c) are inadmissible since number of female operators (*i. e.*, an individual) cannot be in fraction.

Hence, we consider choice (d)

Number of male operators = 10

Number of calls answered by them = 10 × 40 = 400

Total cost = 10 × 250 + 400 × 15 = 8500

$\therefore$ Number of female operators $= \frac{1000 - 400}{50} = 12$

Number of calls answered by them = 12 × 50 = 600

Total cost = 12 × 300 + 600 × 10 = 9600

$\therefore$ total expenditure to answer 1000 calls

= 8500 + 9600 = 18100

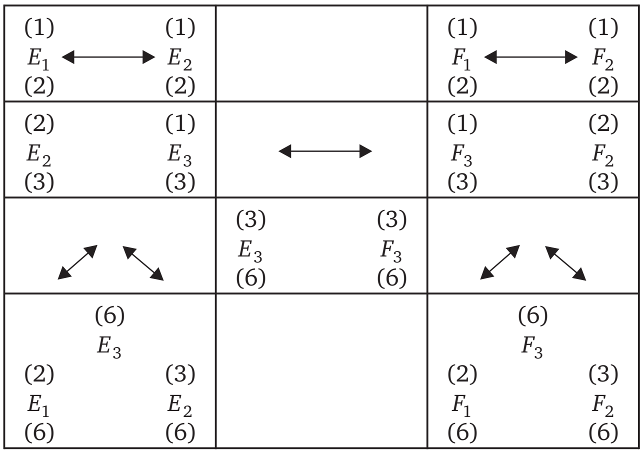
Hence choice (d) is correct.

14. Let E_1, E_2, E_3 be three Englishmen

and F_1, F_2, F_3 be three Frenchmen
and assume that E_3 knows French who can work as mediator between two groups.
The following table shows the probable communication at various steps and the number of secrets known by the individuals at that particular step.

	Step	Communicators	Numbers of Secrets Known
No. of calls	1	$E_3 \leftrightarrow F_1$	$1+1=2$
	2	$E_3 \leftrightarrow F_2$	$2+1=3$
	3	$E_3 \leftrightarrow F_3$	$3+1=4$
	4	$E_3 \leftrightarrow E_1$	$4+1=5$
	5	$E_3 \leftrightarrow E_2$	$5+1=6$
	6	$E_1 \leftrightarrow E_2$	6
	7	$E_3 \leftrightarrow F_1$	6
	8	$E_3 \leftrightarrow F_2$	6
	9	$E_3 \leftrightarrow F_3$	6

Alternatively : The following arrow diagram shows the minimum possible communications.
Number of arrows shows the number of calls. The values written above $E_1, E_2 \ldots$ etc. show the number of secrets known just before the current call and the values written below $E_1, E_2 \ldots$ etc. show the number of secrets known just after the current call.



15. Let the dimensions of each tile be 1×1.
Let us assume that the length and breadth of the room be l and b then

$$\text{total number of tiles} = l \times b$$

and $\quad$ number of red tiles $= (l-2)(b-2)$

but since number of white tiles = number of red tiles

$\therefore \quad lb = 2(l-2)(b-2)$

$\Rightarrow \quad b = \dfrac{4l-8}{l-4}$

Now since l and b both are integers, hence at $l = 12$, we get $b = 5$ an integer.
Hence, choice (b) is correct.

16. See the following pattern.
If $P = 1!$

$\therefore \quad \dfrac{P+2}{2!} = \dfrac{3}{2} \Rightarrow$ Remainder 1

If $P = 1! + 2 \times 2!$

$\therefore \quad \dfrac{P+2}{3!} = \dfrac{7}{6} \Rightarrow$ Remainder 1

If $P = 1! + 2 \times 2! + 3 \times 3!$

$\therefore \quad \dfrac{P+2}{4!} = \dfrac{25}{24} \Rightarrow$ Remainder 1

If $P = 1! + 2 \times 2! + 3 \times 3! + 4 \times 4!$

$\therefore \quad \dfrac{P+2}{5!} = \dfrac{121}{120} \Rightarrow$ Remainder 1

… … …
… … …

Hence we can conclude that the required remainder is 1.
Alternatively : We know that

$$n \times n! = (n+1-1)n! = (n+1)n! - n!$$
$$= (n+1)! - n!$$

Thus we can express

$P = 1! + (3! - 2!) + (4! - 3!) + (5! - 4!) + \ldots (11! - 10!)$

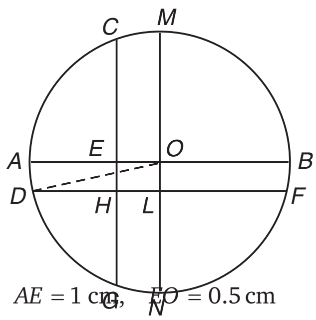
$\Rightarrow \quad P = 1! - 2! + 11! = 11! - 1$

$\therefore \quad P + 2 = 11! - 1 + 2 = 11! + 1$

$\therefore (P+2)$ leaves a remainder of 1 when divided by 11 !

17. $AO = OB = OM = ON = 1.5$ cm

$$AE : EB = 1 : 2$$



$\therefore \quad AE = 1$ cm, $EO = 0.5$ cm

Similarly, $NL = 1$ cm and $OL = 0.5$ cm

$\therefore \quad DL = \sqrt{(DO)^2 - (OL)^2}$

$DL = \sqrt{(1.5)^2 - (0.5)^2}$

$\Rightarrow \quad DL = \sqrt{2}$ cm

$\therefore \quad DH = DL - HL = \sqrt{2} - \dfrac{1}{2} \quad (\because \; HL = 0.5 \text{ cm})$

$\Rightarrow \quad DH = \left(\dfrac{2\sqrt{2}-1}{2}\right)$

18. Let $\quad A = 100m + 10n + p$

$\therefore \quad B = 100p + 10n + m$

$\therefore \quad B - A = 99(p - m)$

$(B - A)$ will be divisible by 7 only when $(p - m) = 7$

$\therefore \quad p = 9, \quad m = 2$ and $p = 8, \quad m = 1$

Please note that p and m can never be zero.
$\therefore$ Possible values of A are 108, 118, 128, 138 … 198, 209, 219, 239, 249,… 289, 299.

Thus choice (b) is most appropriate since other choices do not include some values of A as choice (a) does not include 299.

19. $a_{n+1} = 4n - 2 + 3a_n$

Given $\quad a_1 = 1 = 3^1 - 2 \times 1$

$\therefore \quad a_2 = 4 \times 1 - 2 + 3 \times 1 = 5 = 3^2 - 2 \times 2$

$a_3 = 4 \times 2 - 2 + 3 \times 5 = 21 = 3^3 - 2 \times 3$

$a_4 = 4 \times 3 - 2 + 3 \times 21 = 73 \text{ etc.} = 3^4 - 2 \times 4$

Now, look for the option. In each of the choice either 200 is added or subtracted where 200 implies 100×2 *i. e.*, $n \times 2$ for $n = 100$.
Also ineach of the choice there is either 3^{99} or 3^{100} which implies either 3^{n-1} or 3^n.
Hence by close observation we find that the correct value of $a_n = 3^n - 2n$

Hence $\quad a_{100} = 3^{100} - 2 \times 100 = 3^{100} - 200$

20. There are 3 possible cases.

(i) when odd digits are at 1st and 3rd position and even digits appear at 5th position.
(ii) when odd digits are at 1st and 5th position and even digits appear at 3rd position.
(iii) when odd digits appear at 3rd and 5th position and even digits appear at 1st position. *i. e.*,

	1	2	3	4	5
(i)	0		0		*e*
(ii)	0		*e*		0
(iii)	*e*		0		0

For the first case :

3 2 2 1 2 = 24

For the second case :

3 2 2 1 2 = 24

For the third case :

2 2 3 1 2 = 24

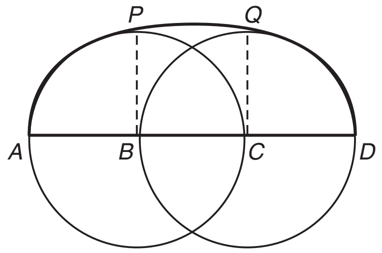
We see that in each case there are 24 numbers.
In the first case at unit place an even digit appears.
Since we know that there are only two even digits (*viz.*, 2 and 4) and 24 numbers, therefore each digit appears $\frac{24}{2} = 12$ times.
Hence sum of the unit digits in this case $= 12(2 + 4) = 72$
In the second case at unit place an odd digit appears. Since we know that there are only 3 odd digits (*viz.*, 1, 3, 5) and 24 numbers, therefore each digit appears $\frac{24}{3} = 8$ times.
Hence sum of the unit digit in this case $= 8(1 + 3 + 5) = 72$
Third case and second case are same. hence in the third case also we get sum of the digits at unit place $= 72$
Thus the total sum of all the three cases $= 72 + 72 + 72 = 216$

21. $(30)^{2720} = 3^{2720} \times 10^{2720}$

unit digit of 3^{2720} is 1
(Refer the chapter of number system)
Hence the required answer is 1.

22. $\quad AB = BC = CD = 1$ m

Also $\quad BP = CQ = PQ = BC$



The path of insect is $APQD$.

$\therefore \quad l(APQD) = l(AP) + l(PQ) + l(QD)$

$l(AP) = 2\pi \times (1) \times \frac{90}{360} = \frac{\pi}{2}$

Similarly $\quad l(QD) = \frac{\pi}{2}$

$\therefore \quad l(APQD) = \frac{\pi}{2} + 1 + \frac{\pi}{2} = (1 + \pi)$ m

23. $\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right) = \log_x x - \log_x y + \log_y y - \log_y x$

$= 2 - (\log_x y + \log_y x)$

But we know that $(\log_x y + \log_y x) \geq 2$

since $\quad \left(a + \frac{1}{a}\right) \geq 2$

$\therefore$ The max. possible value of

$\log_x\left(\frac{x}{y}\right) + \log_y\left(\frac{y}{x}\right)$ is 0

Hence choice (d) is correct.

24. For two digit numbers

$S_n = a + b, \quad P_n = a \times b \quad \text{and} \quad n = 10a + b$

$\therefore \quad S_n + P_n = n$

$(a + b) + ab = 10a + b$

$\Rightarrow \quad b = 9$

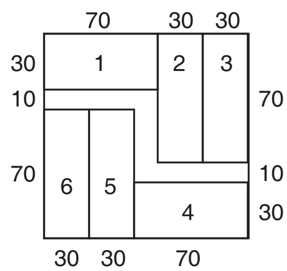
$\therefore$ possible numbers are 19, 29, 39, 49, ... 89, 99.
For three digit numbers

$S_n + P_n = n$

$(a + b + c) + abc = 100a + 10b + c$

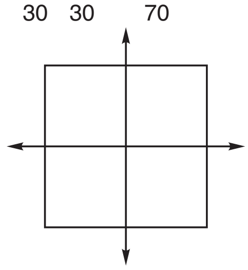
$\Rightarrow \quad abc = 9(11a + b)$

The above equation can not be satisfied for any values of a, b, c.
Hence the total possible numbers = 9



25. Since $\frac{130 \times 110}{70 \times 30} \approx 6.8$

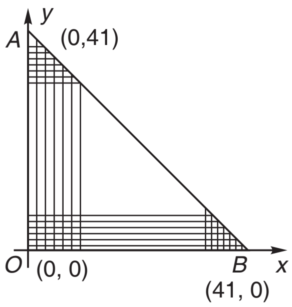
So choice (d) is wrong.
One of the possible arrangement is shown in the figure.



26. There are four cases
(i) $x + y > 0$ and $x - y > 0$

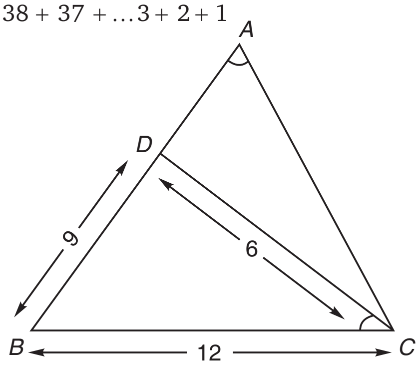
(ii) $x + y > 0$ and $x - y < 0$
(iii) $x + y < 0$ and $x - y < 0$
(iv) $x + y < 0$ and $x - y > 0$
In (i) case $x = 2, y = 0$
In (ii) case $y = 2, x = 0$
In (iii) case $x = -2, y = 0$
In (iv) case $y = -2, x = 0$
$\therefore$ Required area $= 4 \times 4 = 16$

27. We cannot consider the points which lie on the edges of the triangle.

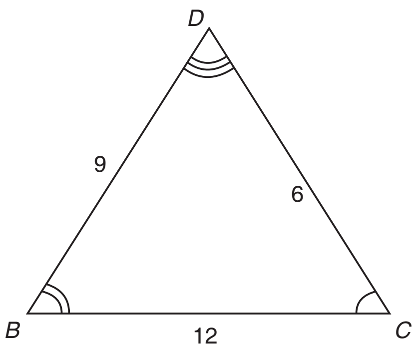


Thus the number of integral points

$= 39 + 38 + 37 + \ldots 3 + 2 + 1$



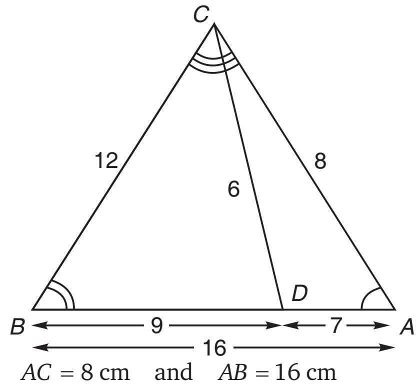
$= \dfrac{39 \times 40}{2} = 780$



28. In the given figures

$\Delta BDC \sim \Delta BCA$

$\therefore \quad \dfrac{BD}{BC} = \dfrac{CD}{AC} = \dfrac{BC}{AB}$



$\therefore \quad AC = 8$ cm and $AB = 16$ cm

$\therefore \quad AD = AB - BD$

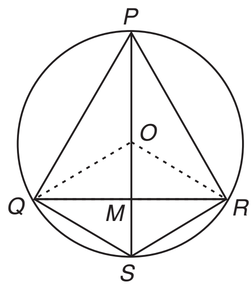
$AD = 16 - 9 = 7$ cm

$\therefore$ Perimeter of $\Delta ADC = 6 + 7 + 8 = 21$ cm

and Perimeter of $\Delta BDC = 9 + 6 + 12 = 27$ cm

$\therefore \quad$ Required ratio $= \dfrac{21}{27} = \dfrac{7}{9}$

29. Let r be the radius of circle.



$\therefore \quad$ Height of $\Delta PM = \dfrac{3}{2}r$

Let a be each side of the triangle

$\therefore \quad h = \dfrac{\sqrt{3}}{2}a$

$\dfrac{3}{2}r = \dfrac{\sqrt{3}}{2}a$

$\Rightarrow \quad a = \sqrt{3}r$

$\therefore \quad PQ = PR = \sqrt{3}r \qquad (\because OM = MS \text{ and } MQ = MR)$

and $\quad SQ = SR = OQ = OR = r$

$\therefore \quad PQ + QS + SR + RP = \sqrt{3}r + r + r + \sqrt{3}r$

$= 2r(1 + \sqrt{3})$

30. The possible numbers are :
1113, 1119, 1131, 1137, 1155, 1173, 1179, 1191, 1197
Thus there are total 9 elements in the set S.