CAT SOLVED PAPER 2006

No. of Questions : 25 *Time :* 40 *min*

✍ ***Directions :*** *This section contains 25 questions. All questions carry 4 marks each. Each wrong answer will attract a penalty of 1 mark.*

1. If $x = -0.5$, then which of the following has the smallest value ?

(a) $2^{1/x}$ (b) $\frac{1}{x}$

(c) $\frac{1}{x^2}$ (d) 2^x

(e) $\frac{1}{\sqrt{-x}}$

2. Which among $2^{1/2}, 3^{1/3}, 4^{1/4}, 6^{1/6}$ and $12^{1/12}$ is the largest ?

(a) $2^{1/2}$ (b) $3^{1/3}$

(c) $4^{1/4}$ (d) $6^{1/6}$

(e) $12^{1/12}$

3. If $\frac{a}{b} = \frac{1}{3}, \frac{b}{c} = 2, \frac{c}{d} = \frac{1}{2}, \frac{d}{e} = 3$ and $\frac{e}{f} = \frac{1}{4}$, then what is the value of $\frac{abc}{def}$?

(a) $\frac{3}{8}$ (b) $\frac{27}{8}$

(c) $\frac{3}{4}$ (d) $\frac{27}{4}$

(e) $\frac{1}{4}$

4. The length, breadth and height of a room are in the ratio 3 : 2 : 1. If the breadth and height are halved while the length is doubled, then the total area of the four walls of the room will :

(a) remain the same (b) decrease by 13.64%

(c) decrease by 15% (d) decrease by 18.75%

(e) decrease by 30%

5. Consider a sequence where the *n*th term, $t_n = \frac{n}{(n+2)}$, $n = 1, 2, \ldots\ldots$

The value of $t_3 \times t_4 \times t_5 \times \ldots\ldots \times t_{53}$ equals :

(a) $\frac{2}{495}$ (b) $\frac{2}{477}$

(c) $\frac{12}{55}$ (d) $\frac{1}{1485}$

(e) $\frac{1}{2970}$

6. A group of 630 children is arranged in rows for a group photograph session. Each row contains three fewer children than the row in front of it. What number of rows is not possible ?

(a) 3 (b) 4

(c) 5 (d) 6

(e) 7

7. What are the values of x and y that satisfy both the equations ?

$$2^{0.7x} \cdot 3^{-1.25y} = \frac{8\sqrt{6}}{27}$$

$$4^{0.3x} \cdot 9^{0.2y} = 8 \cdot (81)^{1/5}$$

(a) $x = 2, y = 5$ (b) $x = 2.5, y = 6$

(c) $x = 3, y = 5$ (d) $x = 3, y = 4$

(e) $x = 5, y = 2$

8. The number of solutions of the equation $2x + y = 40$, where both x and y are positive integers and $x \le y$ is :

(a) 7 (b) 13

(c) 14 (d) 18

(e) 20

9. A survey was conducted of 100 people to find out whether they had read recent issues of Golmal, a monthly magazine. The summarized information regarding readership in 3 months is given below :

Only September : 18; September but not August: 23; September and July : 8;

September : 28, July : 48, July and August : 10;

None of the three months : 24.

What is the number of surveyed people who have read exactly two consecutive issues (out of the three) ?

(a) 7 (b) 9

(c) 12 (d) 14

(e) 17

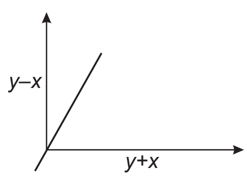
10. The sum of four consecutive two-digit odd numbers, when divided by 10, becomes a perfect square. Which of the following can possibly be one of these four numbers ?

(a) 21 (b) 25

(c) 41 (d) 67

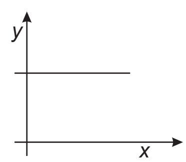
(e) 73

11. The graph of $y - x$ against $y + x$ is as shown below.

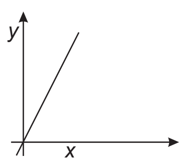


(All graphs in this question are drawn to scale and the same scale has been used on each axis.)

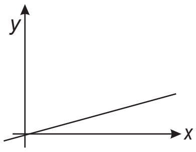
(a)



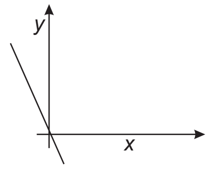
(b)



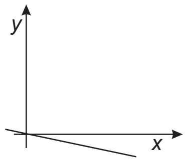
(c)



(d)



(e)



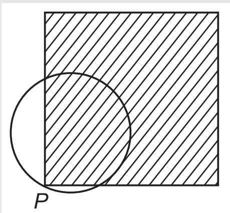
Which of the following shows the graph of y against x ?

12. Consider the set $S = \{1, 2, 3, \ldots\ldots, 1000\}$. How many arithmetic progressions can be formed from the elements of S that start with 1 and end with 1000 and have at least 3 elements ?

(a) 3 (b) 4
(c) 6 (d) 7
(e) 8

✍ ***Directions for Question No. 13 and 14 :*** *Answer Questions 13 and 14 on the basis of the information given below :*

A punching machine is used to punch a circular hole of diameter two units from a square sheet of aluminium of width 2 units, as shown below. The hole is punched such that the circular hole touches one corner P of the square sheet and the diameter of the hole originating at P is in line with a diagonal



of the square.

13. The proportion of the sheet area that remains after punching is :

(a) $(\pi + 2)/8$ (b) $(6 - \pi)/8$
(c) $(4 - \pi)/4$ (d) $(\pi - 2)/4$
(e) $(14 - 3\pi)/6$

14. Find the area of the part of the circle (round punch) falling outside the square sheet :

(a) $\pi/4$ (b) $(\pi - 1)/2$
(c) $(\pi - 1)/4$ (d) $(\pi - 2)/2$
(e) $(\pi - 2)/4$

15. What values of x satisfy $x^{2/3} + x^{1/3} - 2 \le 0$?

(a) $-8 \le x \le 1$ (b) $-1 \le x \le 8$
(c) $1 < x < 8$ (d) $1 \le x \le 8$
(e) $-8 \le x \le 8$

16. Let $f(x) = \max(2x + 1, 3 - 4x)$, where x is any real number. Then the minimum possible value of $f(x)$ is :

(a) 1/3 (b) 1/2
(c) 2/3 (d) 4/3
(e) 5/3

✍ ***Directions for Question No. 17 and 18 :*** *Answer Questions 17 and 18 on the basis of the information given below :*

An airline has a certain free luggage allowance and charges for excess luggage at a fixed rate per kg. Two passengers, Raja and Praja have 60 kg of luggage between them, and are charged. 1200 and Rs. 2400 respectively for excess luggage. Had the entire luggage belonged to one of them, the excess luggage charge would have been Rs. 5400.

17. What is the weight of Praja's luggage ?

(a) 20 kg (b) 25 kg
(c) 30 kg (d) 35 kg
(e) 40 kg

18. What is the free luggage allowance ?

(a) 10 kg (b) 5 kg
(c) 20 kg (d) 25 kg
(e) 30 kg

19. Arun, Barun and Kiranmala start from the same place and travel in the same direction at speeds of 30, 40 and 60 km/h respectively. Barun starts two hours after Arun. If Barun and Kiranmala overtake Arun at the same instant, how many hours after Arun did Kiranmala start ?

(a) 3 (b) 3.5
(c) 4 (d) 4.5
(e) 5

20. When you reverse the digits of the number 13, the number increases by 18. How many other two-digit numbers increase by 18 when their digits are reversed ?

(a) 5 (b) 6
(c) 7 (d) 8
(e) 10

21. A semi-circle is drawn with AB as its diameter. From C, a point on AB, a line perpendicular to AB is drawn meeting the circumference of the semi-circle at D. Given that $AC = 2$ cm and $CD = 6$ cm, the area of the semi-circle (in sq cm) will be :

(a) 32π (b) 50π
(c) 40.5π (d) 81π
(e) undeterminable

22. There are 6 tasks and 6 persons. Taks 1 cannot be assigned either to person 1 or to person 2; task 2 must be assigned to either person 3 or person 4. Every person is to be assigned one task. In how many ways can the assignment be done ?

(a) 144 (b) 180
(c) 192 (d) 360
(e) 716

23. The number of employees in Obelix Menhir company is a prime number and is less than 300. The ratio of the number of employees who are graduates and above, to that of employees who are not, can possibly be :

(a) 101 : 88 (b) 87 : 100
(c) 110 : 111 (d) 85 : 98
(e) 97 : 84

24. If $\log_y x = (a \cdot \log_z y) = (b \cdot \log_x z) = ab$, then which of the following pairs of values for (a, b) is not possible ?

(a) $(-2, 1/2)$ (b) $(1, 1)$
(c) $(0.4, 2.5)$ (d) $(\pi, 1/\pi)$
(e) $(2, 2)$

25. An equilateral triangle BPC is drawn inside a square $ABCD$. What is the value of the angle APD in degrees ?

(a) 75 (b) 90
(c) 120 (d) 135
(e) 150

Answers

1 (b)	**2.** (b)	**3.** (a)	**4.** (e)	**5.** (a)	**6.** (d)	**7.** (e)	**8.** (b)	**9.** (b)	**10.** (c)
11. (d)	**12.** (d)	**13.** (b)	**14.** (d)	**15.** (a)	**16.** (e)	**17.** (d)	**18.** (e)	**19.** (c)	**20.** (b)
21. (b)	**22.** (a)	**23.** (e)	**24.** (e)	**25.** (e)					

Hints & Solutions

1. Given, $x = -0.5 = -\frac{1}{2}$

Now substituting the value of x in each of the five options we get

Choice (1) : $2^{1/x} = 2^{-2} = \frac{1}{4} = 0.25$

Choice (2) : $\frac{1}{x} = -2$

Choice (3) : $\frac{1}{x^2} = 4$

Choice (4) : $2^x = 2^{-1/2} = \frac{1}{\sqrt{2}}$

Choice (5) : $\frac{1}{\sqrt{-x}} = \frac{1}{\sqrt{1/2}} = \sqrt{2}$

Hence, choice (2) gives the least possible value, since it is negative and rest choices give positive values.

2. To compare the given surds (numbers) we have to equate the order of the surds. So we take the LCM of order of surds (*i.e.*, LCM of 2, 3, 4, 6 and 12) and then by adjusting their powers we get as

Choice (1) : $2^{1/2} = 2^{6/12} = (2^6)^{1/12} = (64)^{1/12}$

Choice (2) : $3^{1/3} = 3^{4/12} = (3^4)^{1/12} = (81)^{1/12}$

Choice (3) : $4^{1/4} = 4^{3/12} = (4^3)^{1/12} = (64)^{1/12}$

Choice (4) : $6^{1/6} = 6^{2/12} = (6^2)^{1/12} = (36)^{1/12}$

Choice (5) : $12^{1/12} = 12^{1/12} = (12^1)^{1/12} = (12)^{1/12}$

Since the power (or index) of each resultant number is same so we can compare the base of each number and so the $3^{1/3}$ is the largest number.

3. $\left(\frac{a}{b}\times\frac{b}{c}\times\frac{c}{d}\right)\times\left(\frac{b}{c}\times\frac{c}{d}\times\frac{d}{e}\right)\times\left(\frac{c}{d}\times\frac{d}{e}\times\frac{e}{f}\right) = \frac{a}{d}\times\frac{b}{e}\times\frac{c}{f}$

$\left(\frac{1}{3}\times 2\times\frac{1}{2}\right)\times\left(2\times\frac{1}{2}\times 3\right)\times\left(\frac{1}{2}\times 3\times\frac{1}{4}\right) = \frac{abc}{def}$

$\Rightarrow \frac{abc}{def} = \frac{1}{3}\times 3\times\frac{3}{8} = \frac{3}{8}$

Alternatively : $a : b = 1 : 3$
$b : c = 2 : 1$
$c : d = 1 : 2$
$d : e = 3 : 1$
$e : f = 1 : 4$

$\therefore a : b : c : d : e : f = (1\times 2\times 1\times 3\times 1) : (3\times 2\times 1\times 3\times 1)$
$: (3\times 1\times 1\times 3\times 1) : (3\times 1\times 2\times 3\times 1)$
$: (3\times 1\times 2\times 1\times 1) : (3\times 1\times 2\times 1\times 4)$
$= 6 : 18 : 9 : 18 : 6 : 24$

$\therefore \frac{abc}{def} = \frac{6\times 18\times 9}{18\times 6\times 24} = \frac{3}{8}$

4. Let $l_1 = 3x$, $b_1 = 2x$ and $h_1 = x$
Then area of the 4 walls $A_1 = 2h_1(l_1 + b_1) = 10x^2$

Now, $l_2 = 6x$, $b_2 = x$ and $h_2 = \frac{x}{2}$

$\therefore$ Changed area of 4 walls $A_2 = 2h_2(l_2 + b_2) = 7x^2$

$\therefore$ % decrease in area of 4 walls $= \frac{10x^2 - 7x^2}{10x^2}\times 100$
$= 30\%$

Alternatively : Since we know that in ratio or percentage calculation absolute values never matter.
So we can assume $(l_1, b_1, h_1) = (30, 20, 10)$
$\therefore$ A_1 (Area of 4 walls) $= 2h_1(l_1 + b_1) = 1000$
Again, when $(l_2, b_2, h_2) = (60, 10, 5)$
$\therefore$ A_2 (New area of 4 walls) $= 2h_2(l_2 + b_2) = 700$

Hence, it is obvious that there is 30% decrease in area of the four walls.

5. Substituting the value of $n = 3, 4, 5, \ldots, 53$ in $t_n = \frac{n}{(n+2)}$

Now, $t_3 \times t_4 \times t_5 \times t_6 \ldots t_{51} \times t_{52} \times t_{53}$

$= \frac{3}{5}\times\frac{4}{6}\times\frac{5}{7}\times\frac{6}{8}\times\ldots\ldots\times\frac{51}{53}\times\frac{52}{54}\times\frac{53}{55}$
$= \frac{3\times 4}{54\times 55} = \frac{2}{495}$

6. Let l be the number of children in last row.
Now considering the choices we can have

(1) $l + (l-3) + (l-6) = 630$
$\Rightarrow$ $l = 213$. Hence possible
(2) $l + (l-3) + (l-6) + (l-9) = 630$
$\Rightarrow$ $l = 162$, Hence possible
(3) $l + (l-3) + (l-6) + (l-9) + (l-12) = 630$
$\Rightarrow$ $l = 132$. Hence possible
(4) $l + (l-3) + (l-6) + (l-9) + (l-12) + (l-15) = 630$
$\Rightarrow$ $l = \frac{225}{2}$. Which is not an integral number

Since, number of children can not be in fraction hence it is an impossible value.
(5) $l + (l-3) + \ldots\ldots + (l-15) + (l-18) = 630$
$\Rightarrow$ $l = 99$. Hence possible

Therefore, choice (d) is appropriate.

Alternatively :

Case I : For odd number of rows, let m be the number of students in middle row.

Choice (1) $(m-3)+m+(m+3)=630$

$\Rightarrow$ $m = 210$,. Which is possible

Choice (3) $(m-6)+(m-3)+m+(m+3)+(m+6)=630$

$\Rightarrow$ $m = 126$, Which is possible

Choice (5) $(m-9)+(m-6)+\ldots\ldots+(m+9)=630$

$\Rightarrow$ $m = 90$, Which is possible

Case II : For even number of rows

$$A \quad + \quad B \quad + \quad C \quad + \quad D$$

Choice (2) $(m-4.5)+(m-1.5)+(m+1.5)+(m+4.5)=630$

$\Rightarrow$ $m = \frac{315}{2} = 157.5$

Hence, A, B, C, D are integers. So it is also possible.

$$A \quad + \quad B \quad + \quad C \quad + \quad D$$

Choice(4) $(m-7.5)+(m-4.5)+(m-1.5)+(m+1.5)$

$$+ \quad E \quad + \quad F$$

$+(m+4.5)+(m+7.5)=630$

$\Rightarrow$ $m = \frac{630}{6} = 105$

Therefore the values of A, B, C, D, E and F are non-integers. Hence it is an inadmissible value. Here A, B, C, D, E and F etc are the number of students in various rows.

7. $2^{0.7x}\cdot 3^{-1.25y} = 8\sqrt{6}/27$

$\Rightarrow$ $2^{\frac{7}{10}x} \times 3^{\frac{-5}{4}y} = 2^{7/2} \times 3^{-7/2}$ ...(i)

And $4^{0.3x}\cdot 9^{0.2}y = 8\cdot(81)^{1/5}$

$\Rightarrow$ $2^{\frac{6}{10}x} \times 3^{\frac{4}{10}y} = 2^{3} \times 3^{3/5}$ ...(ii)

Comparing the respective powers (indices) of 2 and 3 in any of the two equations we get $x = 5$ and $y = 2$.

8.
$2x + y = 40$
$2\,(1)+38=40$
$2\,(2)+36=40$
$2\,(3)+34=40$
....
....
$2\,(13)+14=40$

Since $x \le y$, therefore only 13 set of solutions are possible.

Alternatively :

$2x + y = 40$

$\Rightarrow$ $y = 40-2x$

Since $x \le y$

$\therefore$ $x \le 40-2x$

$\Rightarrow$ $3x \le 40$

$\Rightarrow$ $x \le \frac{40}{3}$

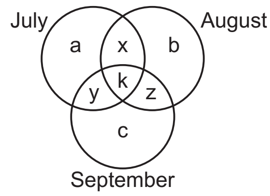
Therefore x can have only 13 positive integers viz 1, 2, 3, ...13.

9. $c = 18$

$y + c = 23 \Rightarrow y = 5$

$y + k = 8 \Rightarrow k = 3$

$c + y + k + z = 28 \Rightarrow z = 2$



$x + k = 10 \Rightarrow x = 7$

$a + x + k + y = 48 \Rightarrow a = 33$

and $(a+b+c)+(x+y+z)+(k)=100-24=76$

$\Rightarrow$ $b = 8$

The number of surveyed people who read exactly two consecutive issues means they read July and August issues and August and September issues only.

Therefore $x + z = 9$. Hence choice (b).

10. Go through options :

Choice (1) $\underline{21}+23+25+27=96$ ×
$19+\underline{21}+23+25=88$ ×
$17+19+\underline{21}+23=80$ ×
$15+17+19+\underline{21}=72$ ×

Choice (2) $\underline{25}+27+29+31=112$ ×
$23+\underline{25}+27+29=104$ ×
$21+23+\underline{25}+27=96$ ×
$19+21+23+\underline{25}=88$ ×

Choice (3) $\underline{41}+43+45+47=176$ ×
$39+\underline{41}+43+45=168$ ×
$37+39+\underline{41}+43=160$ √
$35+37+39+\underline{41}=152$ ×

Similarly we can check out the choice (4) and (5) also. But note that since the desired sum should be a perfect square multiplied by 10. So it has to be zero as the last digit of the sum so we need not to check the choice (2) at all.

Further we need to check only third case of choice (1) Since it has zero (*i.e.*, it is a multiple of 10) So keeping in mind this constraint we need to check just one case in choice (4) and (5) as following

Choice (4) $67+69+71+73=280$ ×

Choice (5) is included in choice (4) itself.

Also note that zero is obtained only when (for odd number) the unit digits of 4 consecutive numbers are 7, 9, 1, 3.

Alternatively : Let $(m-3)$, $(m-1)$, $(m+1)$ and $(m+3)$ be four consecutive odd numbers and n be any natural number then

$(m-3)+(m-1)+(m+1)+(m+3)=10\times n^2$

$\Rightarrow$ $4m = 10n^2$

$\Rightarrow$ $n^2 = \frac{2}{5}m$

Let us consider $n = 1, 2, 3, 4, 5, 6, \ldots\ldots$ etc.

Then at $n = 1, 3, 5, 7$ etc. The value of m is not an integer. Therefore not acceptable.

Now, for $n = 2$, $m = 10$, therefore required numbers are 7, 9, 11 and 13, but these numbers are not available in the choices given.

Again for $n = 4$, $m = 40$, therefore the four numbers can be 37, 39, 41 and 43. Thus we can see here 41 is available in the

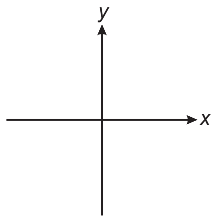


Fig. (i)

option.

Please note that we can check it out for $n = 6, 8, 10, \ldots$ etc. But

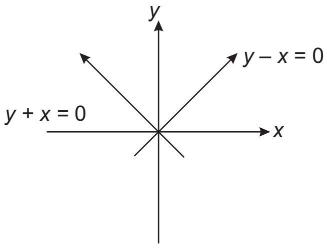


Fig. (ii)

now we don't need to do it.

11. In the coordinate system the equation of X-axis is $y = 0$ and the equation of Y-axis is $x = 0$.

Again the lines represented by $y = x$ $(i.e., y - x = 0)$ and

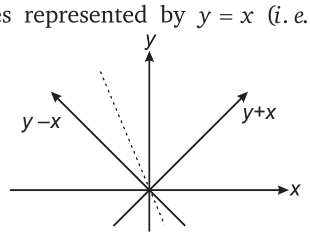


Fig. (iii)

$y = -x$ $(i.e., y + x = 0)$ are perpendicular to each other.

But, here $y - x = 0$ corresponds to the Y+X axis and $y + x = 0$ corresponds to the Y-X axis. Therefore comparing the given figure (of the question) by the above figure (ii) we get the

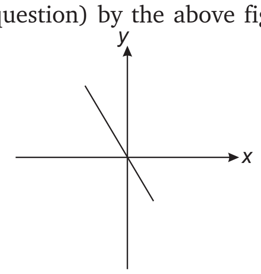


Fig. (iv)

next figure (iii).

Here dotted line represents, the graph shown in the given problem. Hence extracting the requisite information from the above graph we can obtain the following figure.
Hence choice (4) is correct.

Alternatively : The equation of the line graph depicted in the problem can be written as $(y - x) = k(y + x)$, Where k is called as slope (or gradient) of a line measured in anticlockwise. Therefore it is obvious that the angle of inclination is grater than 45° but less than 90°, so $k > 1$; here $k = \tan\theta$.

Now, the above expression $[i.e., (y - x) = k(y + x)]$ can be expressed as

$$y = \left(\frac{1+k}{1-k}\right)x \quad \Rightarrow \quad y = (-m)x$$

[Since $k > 1$, therefore $\frac{1+k}{1-k}$ is a negative value]

Now since 'm' is negative $(i.e., \tan\theta$ is negative) hence θ must be greater than 90° and m is numerically greater than 1.

12. Here first term is $a = 1$ and last term is $l = 1000$. Now, since we know that $l = a + (n-1)d$; where n is the number of terms in progression and d is the common difference

$\therefore \quad 1000 = 1 + (n-1)d$

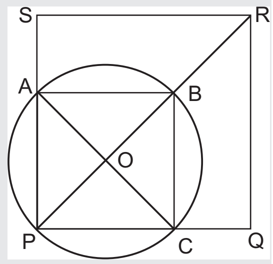
$\Rightarrow \quad 999 = (n-1)d$

Since n and d both are natural numbers therefore $(n-1)$ and d will be the factors of 999. (To find the number of factors study the NUMBER SYSTEM)

$$\begin{aligned}\therefore \quad 999 &= (n-1)\,d \\ &= 1 \times 999 \\ &= 3 \times 333 \\ &= 9 \times 111 \\ &= 27 \times 37 \\ &= 37 \times 27 \\ &= 111 \times 9 \\ &= 333 \times 3 \\ &= 999 \times 1\end{aligned}$$

Since, minimum 3 elements are required, therefore n can not be less than 3 hence $(n-1)$ can not be less than 2. Therefore only 7 values of n (out of 8 values) are possible.

Hence for every value of n we can find an A.P. Hence choice (4) is correct.



✍ ***Solutions for question no. 13 and 14 :***

It is given that the diameter PB coincides with the diagonal PR, of the outer square.

Since the diameter of the circle is 2 unit therefore the area of the circle = π sq unit and the area of the square is 4 sq unit, since each side of the outer square is 2 unit.

NOTE $\angle P$ is 90°, hence AC will behave as a diameter of the circle.

13. Area remained after punching = Area of square $PQRS$ – (Area of semicircle ABC + Area of ΔAPC)

$$= 4 - \left(\frac{\pi}{2} + 1\right)$$

$$= \frac{(6-\pi)}{2}$$

HINT Area of $\Delta APC = \frac{AP \times PC}{2}$; $AP = PC = BC = AB = \sqrt{2}$ unit since $AC = PB = 2$ unit.

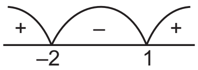
Now the required ratio = $\frac{\text{Area of remaining sheet}}{\text{Area of the original sheet}}$

$$= \frac{(6-\pi)/2}{4}$$
$$= \frac{(6-\pi)}{8}$$

14. Area of the circle falling outside of the square *PQRS*

$$= \text{Area of semicircle } APC - \text{Area of } \Delta APC$$
$$= \left(\frac{\pi}{2}\right) - 1$$
$$= \frac{(\pi - 2)}{2}$$

15. Given that $x^{2/3} + x^{1/3} - 2 \le 0$



$\Rightarrow \quad (x^{1/3})^2 + (x^{1/3}) - 2 \le 0 \qquad$ (Let $x^{1/3} = k$)

$\Rightarrow \quad k^2 + k - 2 \le 0$

$\Rightarrow \quad (k+2)(k-1) \le 0$

$\Rightarrow \quad -2 \le k \le 1$

$\Rightarrow \quad -8 \le x \le 1 \qquad$ (Since $x = k^3$)

(for further clarification, study the QUADRATIC EQUATIONS)

Alternatively : Go through options

Let us consider $x = 8$

$\therefore$ $8^{2/3} + 8^{1/3} - 2 = 4$, Which is not possible since the given expression is less than or equal to zero.

Hence choices (2), (4) and (5) are wrong.

Now consider $x = 1$

$\therefore \quad 1^{2/3} + 1^{1/3} - 2 = 0$

Since, $x = 1$ satisfies the expression, therefore choice (3) is also wrong. Hence choice (1) is correct.

16. Since the given function $f(x)$ contains two line graphs [viz. $(2x+1)$ and $(3-4x)$] which contradicts each other *i. e.*, when one function increases then the other one decreases and vice-versa. Therefore the required value can be obtained when both the functions are equal to each other *i. e*, when $2x + 1 = 3 - 4x$. Actually we can get the required result when both the lines intersect each other.

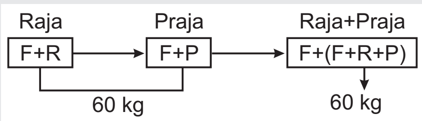
$\therefore \quad 2x + 1 = 3 - 4x$

$\Rightarrow \quad x = \frac{1}{3}$

Now, putting the value of x in the given function we get

$$f(x) = \max(5/3, 5/3)$$

Hence, $\quad \min. f(x) = 5/3$



✍ ***Solutions for 17 and 18*** :

Let $\quad$ *F* → *Free luggage of each one*

R → *Paid luggage of Raja*

P → *Paid luggage of Praja*

Note that for any individual or a group the free luggage allowance is constant which is F only so when Raja and Praja carry the weight individually then each one can avail the free LA (luggage allowance) separately but when they combined their total weights so the one persons's free LA converts into the payble weight, since maximum F kg weight can be carried freely. Therefore,

Raja's payment : Praja's payment : combined payment of Raja and Praja

= R : P : (R + P + F) $\qquad$ *(Here F is the payble weight)*

= 1200 : 2400 : 5400

= $2x : 4x : 9x$

$\therefore \quad (R + P + F) = 9x, \; (R + P) = 6x$

$\Rightarrow \quad F = 3x$

Now, $\quad (F + R) + (F + P) = 60$ kg

$\therefore \quad (3x + 2x) + (3x + 4x) = 60$

$\Rightarrow \quad x = 5$

17. The weight of Praja's luggage = F + P = $7x$ = 35 kg.

18. Free luggage allowance = F = 15 kg for each one but in the absence of exact answer, 30 kg is the best possible answer since it can be assumed that the free luggage of both the persons is being asked.

Alternatively : Let F be the free weight that can be carried by a person or a group combinedely

Now, let W be the weight carried by Raja. So, $(60 - W)$ be the weight carried by Praja.

Again let C be the charges per kg, then

Amount paid by Raja = $(W - F)C = 1200$ $\qquad$ ...(i)

Amount paid by Praja = $[(60 - W) - F]C = 2400$ $\qquad$ ...(ii)

Amount paid by Raja and Praja combined

$= (60 - F)C = 5400$ $\qquad$ ...(iii)

From (i) and (ii)

$$\frac{(W - F)C}{[(60 - W) - F]C} = \frac{1200}{2400}$$

$\Rightarrow \quad 3W - F = 60$ $\qquad$ ...(iv)

From (i) and (iii)

$$\frac{(W - F)C}{(60 - F)C} = \frac{1200}{5400}$$

$\Rightarrow \quad 9W - 7F = 120$ $\qquad$ ...(v)

From (iv) and (v) $F = 15$ and $W = 25$

$\therefore$ Raja's total weight = W = 25 kg and

Praja's total weight = 60 – 25 = 35 kg

And the free luggage of each person = F = 15 kg.

19. Time taken by Barun to overtake Arun

$$= \frac{\text{distance travelled by Arun in 2 h}}{\text{relative speed of Arun and Barun}}$$

$$= \frac{2 \times 30}{(40 - 30)} = 6 \text{ h}$$

It means Arun has to run for (2 + 6) = 8 h

Now since the ratio of speed of $B : K = 4 : 6$

$\therefore$ The ratio of time taken by $\; B : K = 6 : 4$

Therefore it is clear that Kiranmala has to run only for 4 h which in turn implies that Kiranmala has started (8 – 4) = 4 h after Arun.

Alternatively : $\qquad A : B : K$

Speed → $\qquad 3x : 4x : 6x$

Time $\rightarrow$ $\frac{1}{3x} : \frac{1}{4x} : \frac{1}{6x}$

Time $\rightarrow$ $4y : 3y : 2y$

But $4y - 3y = 2 \Rightarrow y = 2$

$\therefore$ Time taken by $A = 4y = 8$ h

Time taken by $B = 3y = 6$ h

Time taken by $K = 2y = 4$ h

Hence Kiranmala has started $8 - 4 = 4$ h after Arun has started.

20. Let a, b be the tens and unit digits of the required two digit number then the number can be expressed as $10a + b$. On reversing the position of the digits we get another number $10b + a$.

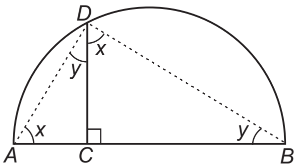
Now, as per the question

$$(10b + a) - (10a + b) = 18$$

$$\Rightarrow 9(b - a) = 18$$

$$\Rightarrow b - a = 2$$

So the possible numbers excluding 13 are 24, 35, 46, 57, 68 and 79. Hence choice (2) is correct.



21. $\angle ADB = 90° = (x° + y°)$

Since $\angle ADB$ is the angle subtended in a semicircle (theorem).

Now, It can be seen from the figure that $\Delta ACD \cong \Delta DCB$

$$\therefore \frac{AC}{CD} = \frac{CD}{BC}$$

$$\Rightarrow \frac{2}{6} = \frac{6}{CB}$$

$$\Rightarrow CB = 18$$

$$\Rightarrow AB = 2 + 18 = 20 \text{ cm}$$

The radius of the semicircle $= \frac{20}{2} = 10$ cm

Hence the area of semicircle $= \frac{1}{2}\pi(10)^2 = 50\pi \text{ cm}^2$

22. The concept incorporated in this problem is very similar to the concept of "posting 6 letters into 6 letter boxes".

Here our job is to assign the tasks to different people.

Task 2 can be assigned in 2 ways-either to person 3 or person 4

Task 1 can be assigned in 3 ways, except the person 1, person 2 and person who has been assigned with the task 2 already.

Now, remaining task 3, task 4, task 5 and task 6 each one can be assigned to the remaining 4 persons in 4! ways.

Hence, the total number of ways in which this can be done $= 2 \times 3 \times (4!) = 144$ ways.

23. Let the required ratio be $m : n$, then $(m + n)$ must be a prime number. Now this can be obtained by checking the options.

Choice (a) $101 + 88 = 189$ which is not a prime number.

Choice (b) $87 + 100 = 187$, which is not a prime number

Choice (c) $110 + 111 = 221$, which is not a prime number

Choice (d) $85 + 98 = 183$, which is not a prime number

Choice (e) $97 + 84 = 181$, which is a prime number

Hence, choice (5) is correct.

24. $\log_y x = ab \Rightarrow y^{ab} = x$ ...(i)

$a \log_z y = ab \Rightarrow \log_z y = b$

$\Rightarrow z^b = y$ ...(ii)

and $b \cdot \log_x z = ab \Rightarrow \log_x z = a$

$\Rightarrow x^a = z$ ...(iii)

From (ii) and (iii) $y = z^b$

or $y = (x^a)^b$

or $y = x^{ab}$

or $y = (y^{ab})^{ab}$ [using eq. (i)]

or $y = (y)^{(ab)^2}$

Comparing the powers (or indices) on both the sides, we get

$$(ab)^2 = 1$$

$$\Rightarrow ab = \pm 1$$

Therefore, choice (5) is not possible.

Alternatively :

$$ab \times ab = (a \cdot \log_z y) \times (b \cdot \log_x z)$$

$$ab \times ab = (ab)(\log_z y \times \log_x z)$$

$$ab = \log_z y \times \left(\frac{1}{\log_z x}\right)$$

$$ab = \frac{\log_z y}{\log_z x}$$

$\left\{\begin{array}{l} \because \log_x y = ab \\ \therefore \log y_x = \frac{1}{ab} \end{array}\right\}$

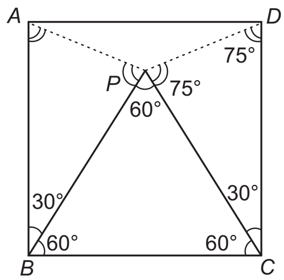
$$ab = \log_x y$$

$$ab = \frac{1}{ab}$$

$$\Rightarrow (ab)^2 = 1$$

$$\Rightarrow (ab) = \pm 1$$

Hence choice (5) is not possible.



25. In equilateral triangle BPC

$\angle PBC = \angle PCB = \angle BPC = 60°$

Now, $\angle PCD = 30° = \angle ABP$;

Since, $\angle B = \angle C = 90°$

Now since all the three sides of ΔBPC are equal to the all the four sides of square $ABCD$. Therefore, $PC = CD$

Hence, $\angle CPD = \angle CDP$ (ΔPCD is an isosceles Δ)

$$\therefore \angle CPD = \frac{180 - 30}{2} = 75°$$

Similarly $\angle APB = 75°$.

(Since ΔABP is an isosceles Δ)

$$\therefore \angle APD = 360° - (75 + 60 + 75)$$

$$\Rightarrow \angle APD = 150°$$