

Insights into Math Concepts

A Bridge between the Concrete and Abstract

"Insights into Math Concepts" was developed to maximize the use of math manipulatives and to bridge the gap between the concrete and the abstract. Rather than focusing on calculations, exercises solidify and extend concepts through series of short, thought-provoking exercises. By building math intuition and number sense, the intent is to make "hard math seem easy" rather than make easy math seem hard.

Math applications were formulated with a number of goals in mind:

- Understanding of underlying concepts rather than repeated drill
- Critical thinking needed for analytical as opposed to mechanical approach
- Attention to detail through periodic use of unpredictable patterns and cumulative skills
- Emphasis on background needed for success in advanced mathematics
- Focus on concepts frequently missed by students with limited math success
- Treatment of objectives targeted in standardized assessment tests
- Systematic series of exercises that require little teacher direction.

Exercises have a simple, uncluttered format. Most students find the steady progression through the various exercises rewarding and motivating. Inviting success and confidence, the material soon nudges them into concepts normally considered quite challenging

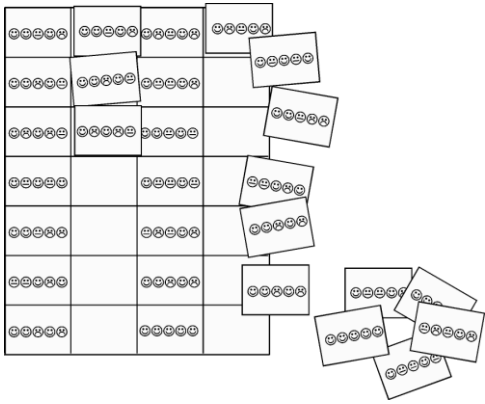
Cardstock Series

Cardstock materials are ideal for individualized progression, resource centers, and cooperative learning applications. Because they are reusable, they provide a cost-effective approach to learning important math skills. Included are cost-effective manipulatives, matching cards, number lines and labels, sequencing cards, and reusable taskcards. Material is color-coded and printed on heavy-weight index or cover stock. All exercises are sized for easy storage in readily available containers or cabinets.

Blackline Series

Blackline masters for extensions and worksheets offer added options. Extensions are variations of matching, sequencing, and number-line exercises. Instead of manipulating small pieces, students write appropriate responses on their own paper. Worksheets are blackline masters of the same taskcards offered in the cardstock series. The blackline series are ideal sequels to the manipulative exercises. They are also valuable when individualized use of cardstock materials is not practical. They are used extensively for diagnostics, whole-class instruction, mastery checks, and homework.

Insights into Math Concepts



Perception Workpages

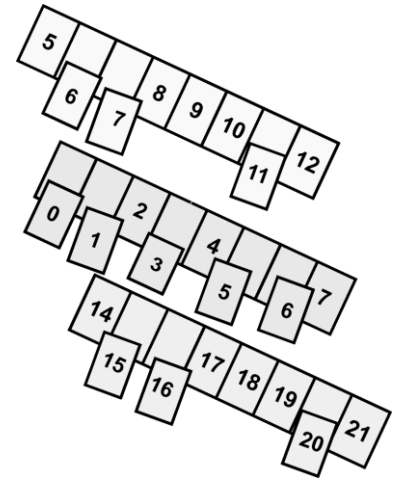
PreK through first grade

Series of graphic representations are paired with identical graphics on full-page taskcards. Each taskcard has 14 "matches." Exercises refine visual perception, left-to-right tracking, and attention to detail. Set of 10 taskcards can be stored in plastic envelopes or sheet protectors, along with appropriate cutouts.

Missing Numbers

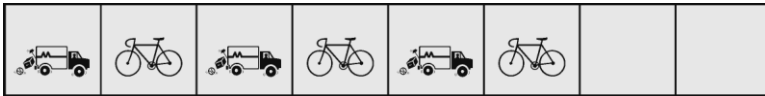
PreK through first grade

Strips that have numerals missing from sequence. Child selects numerals from group of cutouts and places appropriately. Each exercise consists of five sequences. Set includes ten exercises that can be stored in zip lock bags or 5 x 8 inch plastic envelopes. Bags or envelopes then fit nicely into a plastic organizer or index card file box. Blackline extensions are available for children who are able to write.



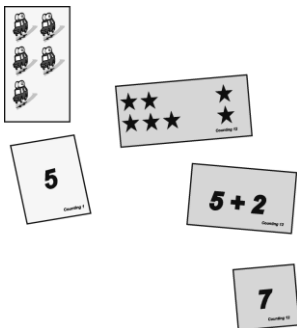
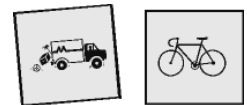
Patterns

PreK through first grade



Cutouts that complete a logical sequence. Each exercise consists of full-page taskcard with five incomplete series. Child selects appropriate

graphic cutouts and completes pattern. Hardest exercises are challenging. Set is comprised of ten exercises that can be stored in plastic envelope or sheet protector, along with appropriate cutouts.



Counting

PreK through first grade

Numerals are paired with appropriate group of pictures. Each exercise deals with numbers from one through twelve. Hardest exercises introduce number sentences with addition and subtraction symbols. Set of fifteen exercises may be stored in 15-drawer hardware cabinet. Blackline masters for extensions are available.

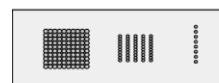
Golden Beads

PreK through second grade

Pictures of golden beads (base ten blocks) that are matched to place value, expanded form, and number words. Young children can also pair with actual golden beads or numeral cards. Each exercise includes six matches. Includes 10 two-digit (blue), 10 three-digit (pink), and 10 four-digit (green). Four-digit exercises can be challenging. 1.75 by 4.25 inch cards. Entire set may be stored in 30-drawer hardware cabinet. Set of ten pages of blackline extensions spanning two through four digits are available.



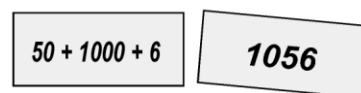
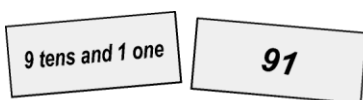
8 tens and 5 ones



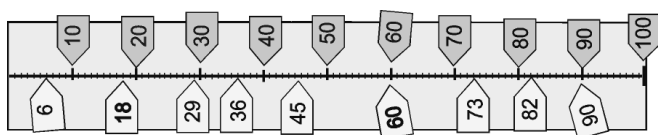
158

Place Value

K- 3rd Grade



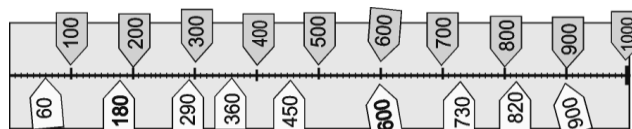
Predictable as well as challenging matching exercises that may be used in conjunction with Montessori manipulatives, particularly numeral cards. Includes place value words, expanded form, and reading numerals. 1.75 x 4.25 inch matching cards may be organized in a variety of ways: 5 to 10 two-digit exercises (blue); 6 to 12 three-digit exercises (pink); 8 to 16 four-digit exercises (green). Store in hardware drawers. Blackline extensions available.



Hundred Line and Labels

K - 2nd Grade

Hundred Line and Thousand Line are similar and can be purchased separately. The purpose is to give the student practice pinpointing random numbers from one to a hundred or one to a thousand. Sets of labels can also be used with appropriate Montessori chains. Labels are arranged in a series of exercises that also help the child, transition between decades/hundreds and round to the nearest ten or hundred. Each set consists of nine exercises that can be stored in hardware drawers. Blackline extensions available.



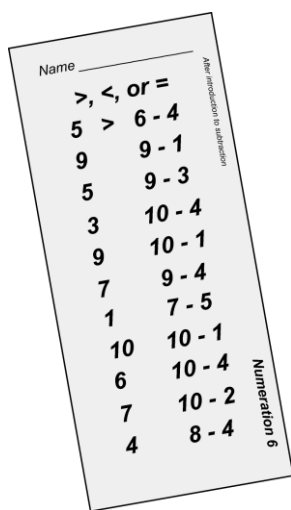
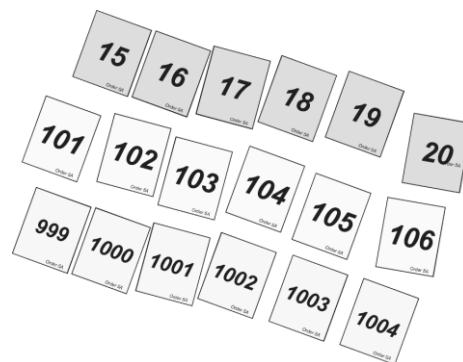
Thousand Line and labels

K - 3rd Grade

Order

K - 3rd grade

1.75 x 2 inch cards that can be manipulated to sequence numerals. Each exercise consists of four sequences of comparable difficulty. Concepts include sequencing by increments of one; even and odd; transitioning to the next decade, hundred, or thousand; and sequencing random numerals from least to greatest. 15 two-digit exercises (blue) for; 8 three-digit exercises (pink); 8 four-digit exercises (green). Entire set may be stored in 30-drawer cabinet. Blackline extensions available.



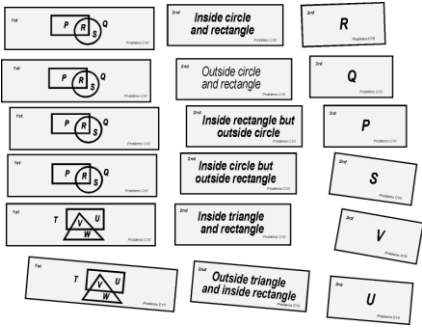
Numeration (Blue)

K - 3rd Grade

Free response task cards requiring application of numeration concepts involving one and two-digit numbers: odd and even, relationship symbols, number patterns, ordering, rounding to nearest ten. Use with Montessori manipulatives encouraged for younger students. Can be used as abstract work for older students. Includes answer keys so child may self-check work. Set is comprised of 18 one-third page task cards which may be stored in 9 x 2.5 inch plastic organizer. Also available in blackline masters.

Problem Solving Levels A, B, and C

K - 4th grade

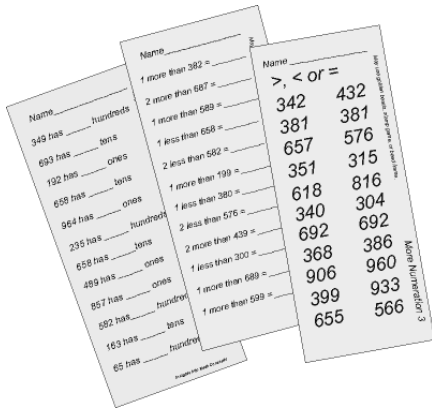
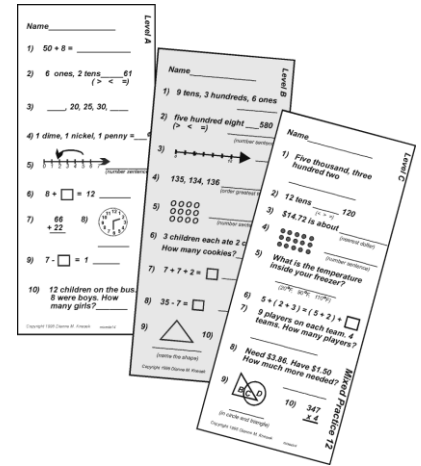


Compilation of concepts requiring critical thinking skills: place value, expanded form, patterns, rounding, applications, equations from pictures/number lines, vocabulary, single and multistep word problems, time, money, fractions, estimating, properties, geometry, and logic. Each exercise addresses a separate concept. Teacher may organize as either matching or free response exercise. Answer sheets included. Great to prepare for standardized tests. Each level has fifteen exercises. May be stored in hardware drawers. Blackline extensions available.

Mixed Practice Levels A, B, and C

K - 4th grade

Mixed skills on free response task cards encompassing numeration, equations with unknowns in any position, operations, time, money, fractions, geometry, measurement, reasonableness, word problems. Third-page free response task cards may be stored in 9 x 2.5 inch plastic organizer. Answer sheets included. Each level is comprised of 15 taskcards. Also available as blackline masters.



More Numeration (Pink)

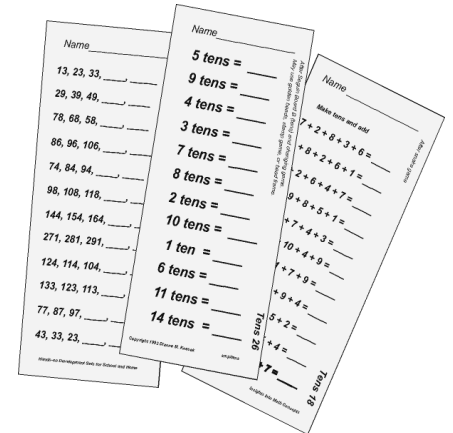
Grades 1 - 3

Numeration concepts involving three-digit numbers: relationship symbols, place value, expanded form, "before and after," number patterns, reading numerals, ordering, rounding to the nearest hundred. Answer sheets included so child may self-check his/her work. Third page free response taskcards may be stored in 9 X 2.5 inch plastic organizer. 15 taskcards. Also available as blackline masters.

Tens (Gold)

Grades 1 - 3

Children learn that ten is a "magic number" and can make operations easy. Ten as an addend and subtrahend, nine as an addend, increasing or decreasing in ten's place, making tens to add, number patterns, multiples of ten, rounding. Third page free response task cards may be stored in 9 x 3 inch plastic organizer. 30 taskcards and answer sheets. Also available as blackline masters.

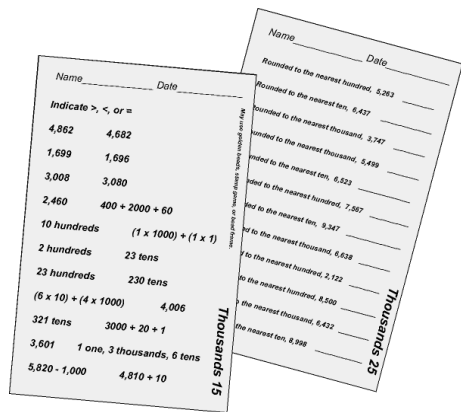


Exchange

Grades 1-3



Renaming groups of ten ones, ten tens, or ten hundreds. Introduces regrouping notation for addition and subtraction. Young students do with manipulatives; older children may do as abstract exercise. Hardest exercises are challenging. Two and three part matching cards which may be stored in hardware drawers.



Thousands

Grades 2-4

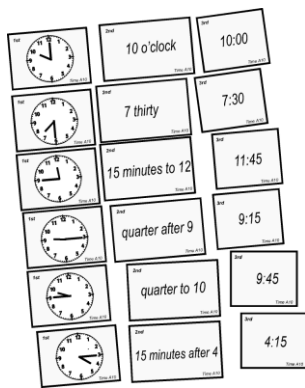
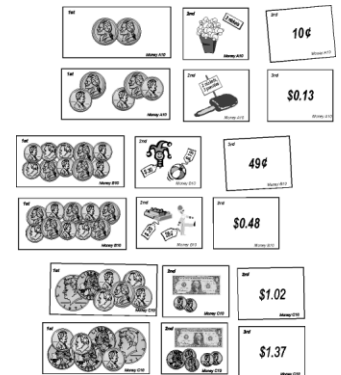
Challenging sequence of exercises exploring 4-digit numeration concepts: relationship symbols, place value, expanded form, "before and after," number patterns, zero in minuend, reading numerals, ordering rounding to nearest thousand, hundred, and ten. Half-page taskcards and answer sheets. 25 exercises. Also available as blackline masters.

Money Match

Levels A, B, C

Grades 1-4

Three-part matching exercises covering money concepts. Level A includes identification of all coins, counting quantities with dimes and pennies, introduction of nickels as well as dollar, five-dollar, ten-dollar, and twenty-dollar bills. Level B emphasizes larger number of nickels, alone and in combination with dimes and pennies. Also includes equivalent groups of coins, association of coins and price tags, as well as change from one quarter, half dollar, or a dollar bill. Level C involves a large number of coins and bills, pricing a combination of items, and more challenging change exercises.



Time Match

Levels A, B, C

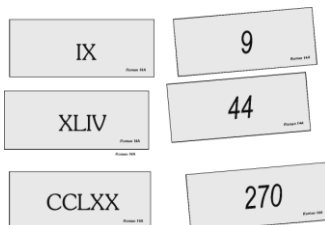
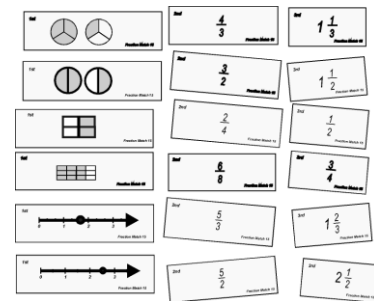
Grades 1-4

Systematic series of matching exercises for reading analog clocks. Difficulty increases in successive levels. Level A reinforces time to the hour, half hour and quarter hour. Level B involves time in five-minute intervals, as well as a given number of hours before or after. Level C reinforces time to the minute as well as time intervals involving minutes before or after a given time. 10 exercises in each level. Blackline extensions also available.

Fraction Match

Grades 2-4

Matching cards involving fractions with a numerator of one, fractions with numerator greater than one, fraction of a set, improper fractions, mixed numbers, fractions on a number line, equivalencies, and simple addition of fractions. Good bridge between fraction manipulatives and "Fraction Concepts." Fifteen exercises in set; blackline extensions also available.



Roman Numerals

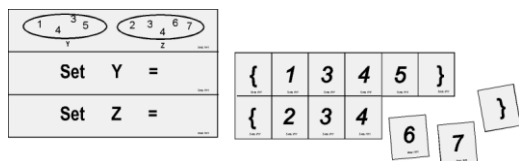
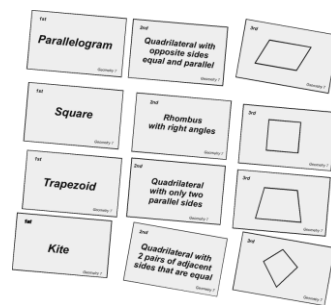
Grades 3 - 5

Systematic series of matching exercises for learning Roman numerals from 1 - 3000. Set of 15 exercises for moderate pace; 30 exercises for slower pace. Answer sheets included. May store in 15 or 30-drawer hardware cabinet. Blackline extensions also available.

Geometry Introduction

Grades 3 - 5

Three-part matching cards reinforcing basic geometric concepts: undefined terms (e.g., point, line, ray); types of lines; angles; plane figures; polygons; triangles; quadrilaterals; perimeter; area, circles; solid figures; volume. Fifteen exercises and answer sheets. May store in 15-drawer cabinet. Blackline extensions also available.



Sets (Introduction)

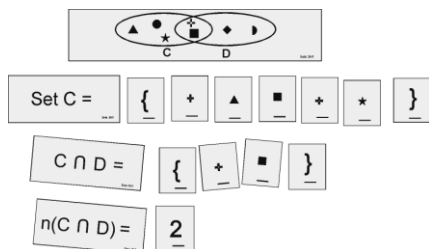
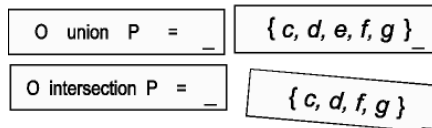
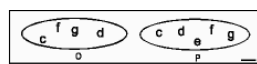
Grades 3-6

"Fun" introduction to set theory. Small cards that can be manipulated to learn set notation, term "element," empty set, and cardinal number. Most teachers organize to form about nine separate exercises. Answers included. May be stored in 4 x 8 inch zip bags (included) or vinyl envelopes.

Sets (Union & Intersection)

Grades 3-8

Continuation of set theory on previous page. Matching exercise on set notation, union & intersection, and Venn Diagrams. Answers included. May store in zip bags or vinyl envelopes.



Sets (Cumulative)

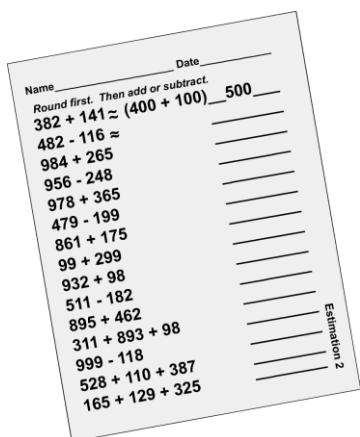
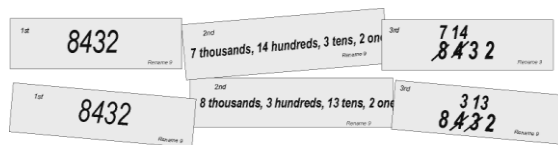
Grades 4 - 8

Small cards that be manipulated for cumulative exercises on set theory: elements, cardinal number, empty set, union, intersection, Venn diagrams. Older students can do as free response exercise. Answers included. May be stored in 4 x 8 inch zip bags (included) or vinyl envelopes.

Rename

Grades 4-6

Challenging exchanging exercises involving multiple regroupings, with and without zeroes as place holders. Two and three-part matching cards that can be stored in hardware drawers. Answers included. Ten exercises.



Estimation

Grades 4 - 6

Free-response exercises on estimation of sums, differences, products, and quotients involving whole numbers, mixed numbers, and dollars/cents. Concepts heavily emphasized by National Council of Mathematics and standardized tests. Ten full-page task cards and answers. Store in large sorter or portable file. Also available in blackline masters.



Advanced Numeration

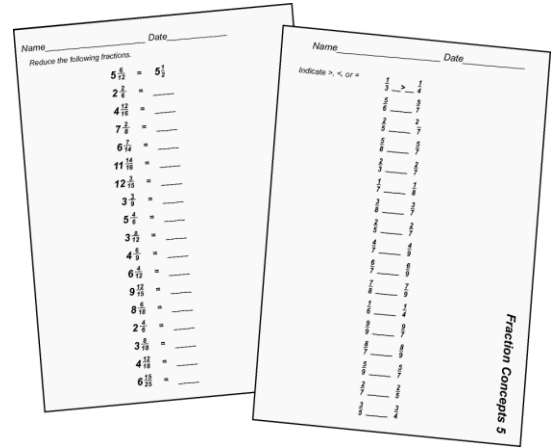
Grades 4-6

Full-page exercises involving numbers up to nine digits: place value, expanded form, comparisons, adding powers of ten, reading large numerals, rounding, cumulative reviews. 20 free-response taskcards and answer sheets. Store in large plastic sorter or portable file. Also available as blackline masters.

Fraction Concepts

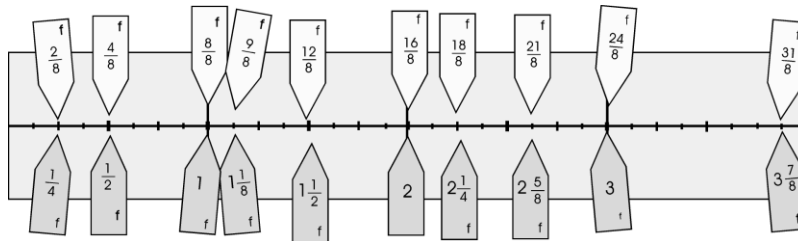
Grades 3-8

Introduction of key fraction concepts: equivalency, comparisons, reducing, improper fractions to mixed numbers, mixed numbers to improper fractions, renaming, fraction of a group, addition and subtraction with the same denominator. Young students use in conjunction with Montessori fraction materials. Older students may do abstractly. 20 full-page taskcards and answer sheets. Store in large plastic sorter or portable file. Also available as blackline masters.



Fraction Number Lines and Labels

Grades 4-8

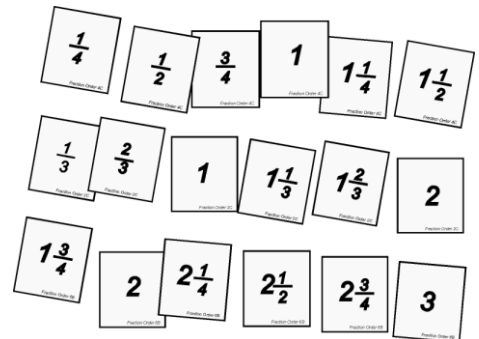


Helps the child visualize the relationship between proper fractions, improper fractions, and mixed numbers and acquire an intuitive understanding of their value. Also aids understanding of linear measurement to sixteenths of an inch. Included are ten color-coded number lines divided into various fractional parts. For example, a bright green line is divided into halves; pink lines are divided into thirds, etc. A set of ivory labels represents proper and improper fractions; white labels include simplified mixed numbers. As an extension, students may place fraction labels on the decimal line (p. 8). Blackline masters for labels are included for replacement parts. Store labels in ten hardware drawers. Blackline extensions also available.

Order Fractions

Grades 4 - 8

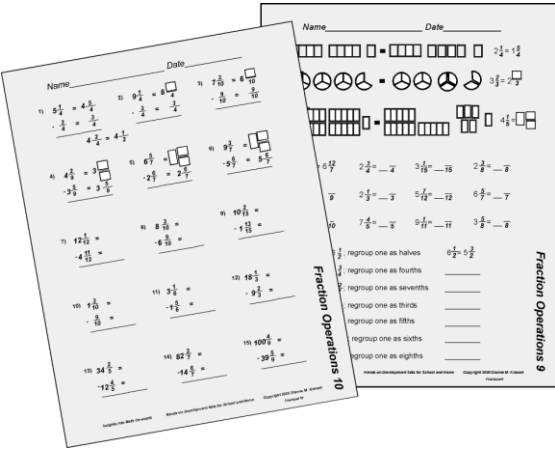
Ordering fractions from least to greatest; includes like denominators, like and unlike fractions, improper fractions, mixed numbers. Each exercise consists of 4 sequences, with 6 fractions per sequence. Ten exercises, comprised of 1.75 x 2 inch cards, may be stored in hardware drawers. Answer sheets included. Blackline extensions also available.



Fraction Operations

Grades 4 - 8

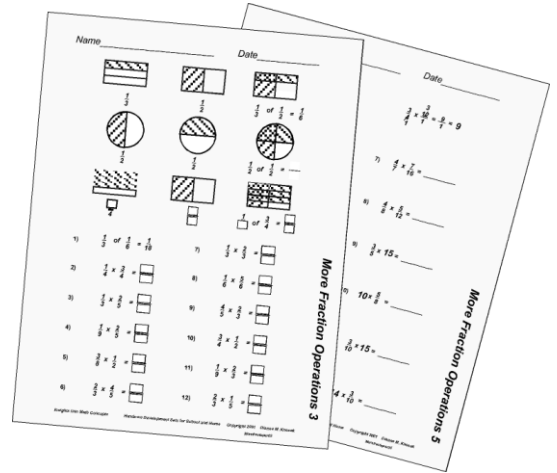
Full-page taskcards pick up where "Fraction Concepts" left off. Step-by-step sequence of skills required in addition and subtraction of unlike fractions: common denominators, addition & subtraction of unlike fractions, unlike mixed numbers, regrouping of fractional minuend, multistep subtraction, and mixed addition and subtraction of fractions and mixed numbers. 13 taskcards and answer sheets. Store in large plastic sorter or portable file. Also available as blackline masters.



More Fraction Operations

Grades 4 - 8

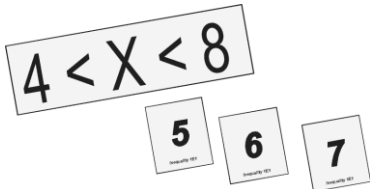
Full-page taskcards refining multiplication and division of fractions, whole numbers, and mixed numbers. Culminates in several mixed exercises requiring all four fraction operations. 20 taskcards and answer sheets. Store in large plastic sorter or portable file. Also available as blackline masters.



Inequalities

Grades 4 - 8

Comfortable introduction to algebraic notation. Also develops logic and critical thinking skills. Algebraic representations of open and closed intervals are solved by selecting cards from a given replacement set. Introduces variables as well as symbols $>$, \geq , $<$, and \leq . Format is 1.75 X 2 inch cards that can be manipulated. Solution cards are 1.5 by 1.75 inches. Store in hardware drawers. Twelve problems per exercise; 12 exercises with answers in the entire set.



Number Theory

Grades 4 - 8

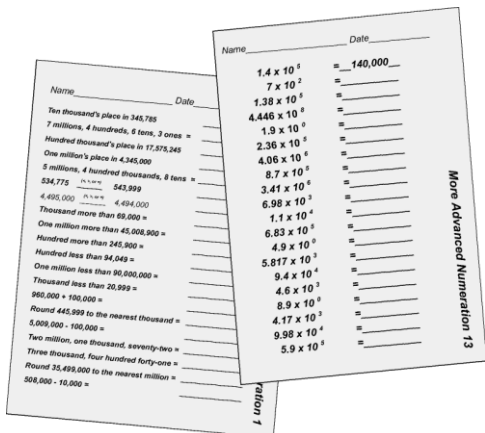
Full-page taskcards on number theory: prime and composite numbers, laws of divisibility, factors, prime factors, greatest common factor, multiples, least common multiple, exponents, simple radicals. 20 taskcards and answer sheets. Store in large plastic sorter or portable file. Also available as blackline masters.



More Advanced Numeration

Grades 5-8

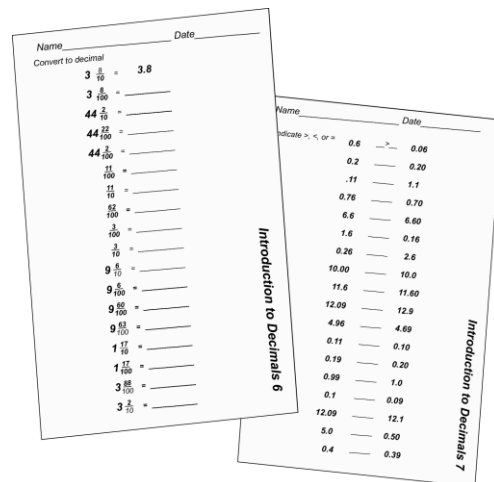
Challenging full-page numeration taskcards for the older student: place value of large numbers, expanded form, powers of ten, decimal concepts, scientific notation with positive and negative exponents. 20 taskcards and answer sheets. Store in large plastic sorter or portable file. Also available as blackline masters.



Decimal Introduction

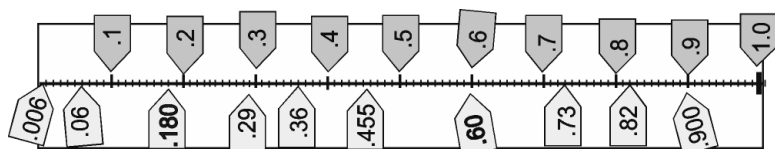
Grades 4 - 8

Series expanded from 10 to 20 full-page taskcards. Full-page taskcards focusing on one and two-place decimals: fraction to decimal, decimal to fraction, comparisons, rounding, and simple operations. Store in large plastic sorter or portable file. 20 exercises and answer sheets. Also available as blackline masters.



Decimal Line and Labels

Grades 4 - 8



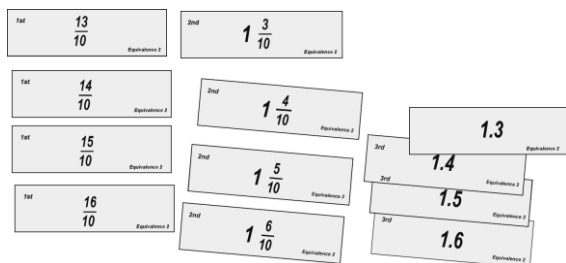
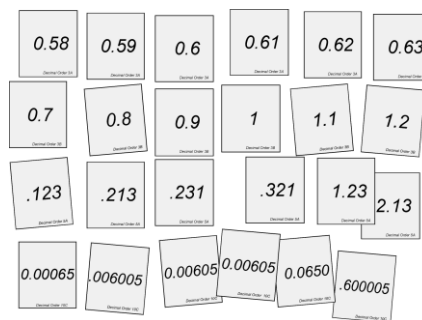
Two "units" divided into hundredths and thousandths respectively. Beginning students label the line divided into hundredths; older students label the line divided into thousandths. Helps students visualize the differences in decimals (for

example 0.9, 0.09, and 0.009). As an extension, fraction labels (detailed on p. 7) may also be used on the decimal line. Blackline masters of labels included for replacement parts. Nine exercises for each line: may be stored in hardware drawers. Blackline extensions also available.

Order Decimals

Grades 4 - 8

Ordering decimals with varying number of places from least to greatest. Hardest exercises are challenging. Skill stressed by National Council of Teachers of Mathematics. 1.75 x 2 inch cards may be stored in hardware drawers. Each exercise has four sequences of six. 10 exercises with answers. Blackline extensions also available.



Decimal/Fraction Equivalence

Grades 5 - 8

Three-part matching cards that match fraction and decimal equivalents, with emphasis on commonly used percents. Older children do abstractly and use "match" as a check. May store in hardware drawers. Eight problems per exercise. Ten exercises with answer sheets. Blackline extensions also available.

Percent

Grades 5 - 8

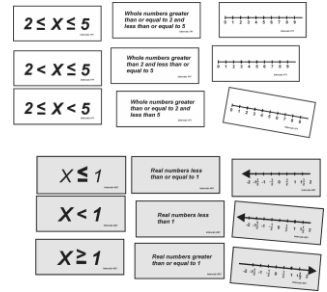
Free-response exercises that introduce concept of percent and encourage intuition in its application. First 10 exercise emphasize fractional equivalents through conversions, percent of a number, estimation, probability, tax, and shipping costs. Second half of the set introduces decimal conversions, comparison of values in different formats, percent of a number, simple interest, selecting easier approach to applications, word problems, and cumulative exercises. 20 exercises in set. Also available as blackline masters.



Intervals

Grades 5 - 8

Verbal, algebraic, and number line expressions depicting open and closed intervals. "Wholes" focuses on discrete whole numbers; "Positive Reals" stress continuous points including fractions; "Integers" include discrete positive and negative integers; "All Reals" emphasize continuous positive and negative values, including fractions. Younger students may do as match; older students do abstractly and use "match" as a check. May be stored in hardware drawers. 10 exercises with answer sheets in each set. Blackline extensions also available.



negative integer, for that reason, a number line or integers has an arrow.

Positive and negative numbers may be written as positive 4, -4 or $^{-}4$.

Name _____

Using exponents with negative numbers:
Do calculations inside the () or | first, front of the () or | last.

Tell whether the following

1) Positive 2 _____ $2^2 = 2 \times 2 = 4$ $(^{-}2)^2 = 4$
 4) $^{-}19$ _____ $(^{-}19)^2 = 361$
 7) $+10,235$ _____ $10^2 = 100$

Which is more?

10) 5 or $+4$ _____ $(^{-}2)^2 = 4$
 13) 0 or -1 _____ $\frac{^{-}16}{3^2} = \frac{^{-}16}{9}$
 >, <, or = _____ $|^{-}3 \cdot 2| = 6$
 16) -3 _____ 0 _____ $(4 \cdot 3)^2 = 144$

Operations with Negatives

Grades 6 - 8

Introduction of negative integers and rationals with number line and counters. Addition, subtraction, multiplication, and division with counters and number line. Division as inverse of multiplication. Absolute value, rules for all four operations, logic, order of operations, mixed practice, and word problems. 20 free-response task cards. Also available as blackline masters.

Order of Operations

Grades 6 - 8

Order of operations based on absolute value, grouping symbols, exponents, multiple grouping symbols, mixed operations without grouping symbols, variables representing specific values, numerators and denominators, and word problems. 10 free response taskcards. Also available as blackline masters.

13) $\frac{5(2+3^0+6)}{3^2-2 \cdot 3}$ _____ 14) _____

Evaluate the following if $m = 2$, $n = 1$, and $p = 3$:

15) $m + n \cdot p =$ _____

16) $n[m p (1 - n^0) - 3] =$ _____

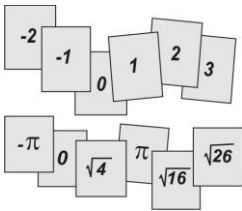
17) $\frac{p^2 - |m + n \cdot 2|}{mnp^2} =$ _____

18) Add 3 and 2. Square the result and then add 4.
 $[(3 + 2)^2 + 5] + 4 =$ _____

Order Reals

Grades 6 - 8

Sequencing integers with emphasis on negatives. Last several exercises feature sequencing positive and negative fractions, decimals, and radicals as well. Each exercise consists of four sequences of six. Ten exercises and answer sheets in the set. Store in hardware drawers. Blackline extensions also available.



Algebra Concepts

Grades 6 - 9

Prerequisites for solving algebraic equations: Classification of numbers as reals, rational or irrational, integers, whole numbers, and/or natural numbers. Variables, constants, verbal and algebraic equivalents, and computing value of algebraic expressions. Distributive property, combining like-terms, multiplying and dividing terms with exponents. 10 free-response task cards. Also available as blackline masters.

3) 0.3 R _____ Q _____ I _____ Z _____

Consider the expression $4a^2 + 5b + 8$.

5) How many terms are there? _____ 6) List the terms _____
 7) What are the constants? _____ 8) List the variables _____
 9) List the exponents. _____ 10) What are the coefficients? _____

If $x = 3$, $y = -2$, and $z = 1$, find the value of each of the following:

11) $x^2 + y^2 - yz =$ _____ 12) $x^2(y - yz) =$ _____

Write a verbal expression for the following algebraic expressions:

14) $\frac{cd}{4}$ _____
 15) $5(a + b)$ _____

Write an algebraic expression for the following:

16) Eight more than two times the square of some number _____
 17) Five less than the ratio of p to q _____

Simplify by distributing or dividing to remove the parentheses:

1) $2y^2 - 3(y - 2) + y - 4$ _____ 2) $4(3 - 5) - 2(1 - 3)$ _____



Solve and check the following equations on separate paper.

6) $y + 3.4 = 2(5)$ 7) $p - \frac{1}{2} = \frac{1}{4}$

9) $5x = 12$ 10) $3x + x = 24$

12) $2(2q - 1) = q + 1$ 13) $\frac{5x - 2}{3} = 6$

15) Based on the function rule (equation), input values for x and y value. Then plot the function on the graph.

$y = 2x + 1$

x	y

What are the x-intercepts? _____
 What is the y-intercept? _____
 What is the slope? _____

Algebra Equations

Grades 6 - 9

Concept of equations, properties of equality, testing solutions, one-step equations involving addition, subtraction, multiplication, and division properties. Multi-step equations with variables on one side as well as on two sides. Solving proportions algebraically. Introduction of functions and linear equation graphs. 15 free-response taskcards. Also available as blackline masters.



